

# Coordinates

Volume XVIII, Issue 9, September 2022

THE MONTHLY MAGAZINE ON POSITIONING, NAVIGATION AND BEYOND



SatNaav is just an enabler for autonomous vehicles - Kym Osley

Fully autonomous shipping is coming - Hesham M. Helal

Role of the marine navigator will change - John Johnson-Allen

AI does not replace a GNSS professional - Jiří Svatoň



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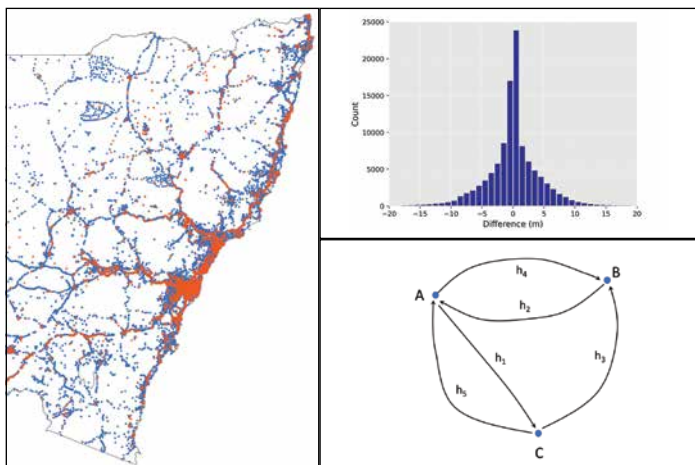


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5G is set to roll out in India very soon,

With speed up to 10 Gbps – supposedly ten to hundred times more

Said to be game a changer

Especially in the sectors like education, health, finance, transport, agriculture...

However, there are some concerns and challenges...

Needful infrastructure, especially in tier 2 and tier 3 cities, rural areas,

Technological readiness and upgrade of the existing devices,

Affordability of the services,

Interference, if any, with the aircraft navigation system, radiation...

So more speed, better service, more money, ...

Groundwork for 6G in due course,

And more meaning to life. Is it?

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# “Situational and spatial awareness is still very much a human activity”

*says Air Vice-Marshal Kym Osley AM, CSC, FAIN, Executive Secretary, Australian Institute of Navigation (AIN)*



**Air Vice-Marshal  
Kym Osley**

Air Vice-Marshal (AVM) Kym Osley joined the Air Force in 1977 and flew as an aerial navigator in F-111, Phantom, F-18 and other fast jet aircraft. In the 1990s, AVM Osley was responsible for major strike-reconnaissance projects within Capability Development (Air) Branch before becoming the Air Force strategic planner for three years and then commanding No 1 Squadron. In 1999, AVM Osley was posted to the United Kingdom as the Air Force Adviser before being appointed Officer Commanding No 82 Wing. In August 2004 he was promoted to Air Commodore and assisted with planning the future force structure for the Australian Defence Force.

AVM Osley deployed as Director of the Coalition Air Operations Centre in the Middle East in 2006-07 where he directed the employment of 425 Coalition aircraft and 25,000 airmen. After commanding Air Combat Group in 2007-08, he was promoted to AVM as Head of Australian Defence Staff (Washington) and then led the Australian Joint Strike Fighter Program from late 2010. In 2014, AVM Osley transitioned to the Reserves and took up a full-time position as a Managing Director in PricewaterhouseCoopers for six years, including leading the team to setup Defence Space Command.

**What are the objectives of Australian Institute of Navigation (AIN)? Would you like to highlight some recent activities and the achievements of the AIN?**

The Australian Institute of Navigation celebrated its 72nd anniversary this year, and continues to fulfil its key roles in promoting excellence in the application of spatial awareness technologies, advocating for leading edge developments in the field of navigation and recognising excellence among military and commercial practitioners of Navigation. Our members include those who have contributed significantly to the advancement of navigation in both the military and commercial arenas, including leading edge thinkers in the field of satellite navigation. The ‘best of the best’ military navigation practitioners in the RAN and RAAF are recognised through annual awards of the Australian Institute of Navigation. In recent times, the AIN has partnered with military and Government agencies to explore how the risks associated with potential interruptions to satellite navigation can be mitigated. The AIN was a key advocate for the introduction of a Space Based Augmentation System in Australia.

**You have over 40 years of experience in defence especially in air defence. You have closely observed and used the navigation technology. In your perspective, what is the general scenario of navigation technology in terms of status, trends and challenges?**

The challenge in military combat aircraft is no longer monitoring (or ‘babysitting’) navigation systems and trying to determine where you are. The ability to know where you

are with high accuracy at all times is now a given. The new challenge is the very high volume of navigation, situational awareness, sensor and other relevant military data that is bombarding the humans-in-the-loop of modern weapon systems and those individuals being able to assimilate data relating to the entire battlespace, validate the data when the enemy is trying to inject uncertainty and respond with the right decision, the right action at exactly the right time to achieve an overall outcome. Situational and spatial awareness is still very much a human activity – it is certainly not just about technology. The best, most capable and well-trained individual will prevail – not the person with just the best absolute technology. Thus all of the leading edge technology that is now entering the military eco-system is causing massive changes to the training and responsibilities of war fighters. A very large future challenge for navigation/situational awareness technology is the human/group interface – the information cannot just be displayed...it will need to be integrated in near real-time and aligned with real-time decision making of individuals and groups to be truly advantageous in getting a decisive combat outcome.

### **What is the status of satellite infrastructure and navigation technologies like Satellite-Based Augmentation Technologies (SBAS) and National Positioning Infrastructure Capability (NPIC) in Australia?**

In the past 75 years, Australia has been at the forefront of many navigation developments but the implementation of those technologies has sometimes been slow due to having a large country with a relatively small population. SBAS is a case in point, where the benefits of SBAS were widely recognised in Australia but we were one of the last nations to get SBAS due to challenges in funding a national system. This remoteness may also delay the introduction of local high accuracy

navigation augmentation systems in Australia for industry, transport and other applications when compared to other more populated regions around the globe. An area we are likely to see considerable advance is in urban air mobility and logistics resupply by uninhabited air systems – once the issues with civil air regulations are overcome.

### **Recently, Australia has also established National Space Agency. How is this development has helped the user communities?**

While the Australian Space Agency is still a very new organisation, it has provided a centre of gravity for Government and Industry to determine broad strategies for how Australia should move forward at a national level in the space domain. It has provided a means for Government to start to articulate where it sees the priorities for development of national space capabilities and where the Government may provide seed funding for Australian space industry. It has also provided a central area to coordinate space-related legislation and regulations, which will be essential enablers for future Australian space enterprises. At this time, it is a modestly sized organisation that is heavily focussed on advocating for space-related industry and supporting the development of a space regulatory framework. In time we will see if the people and funding are provided to create a more comprehensive National Space Agency.

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This mutual reliance between GNSS and other technologies (including mobile technologies and 5G) to deliver overall spatial awareness, information and communications services will grow over time, but become less obvious to the users who are just interested in the service being provided

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## **This is a world of multi-GNSS systems. What advantages do you see about this scenario?**

I think we can expect nations to continue to want to field GNSS systems – for many reasons including national prestige, reliability of service, higher capabilities, etc. With ever cheaper launch costs and much increased use of Low Earth Orbit satellites we can expect that GNSS systems may also increasingly be fielded by non-state providers as a service. From a user perspective, multi-GNSS capable receivers will likely become almost ubiquitous, and offer greatly increased reliability and accuracy.

## **With increasing dependence on GNSS, how do you perceive the threats like interference, jamming and spoofing?**

In a lifetime, we have gone from no GNSS to a high reliance on GNSS for positioning, navigation and timing (PNT). Interestingly, it would appear that because military and commercial aircrew train in so many reversionary navigation and flying modes, aviation is likely to be among the least affected areas due to interference, spoofing and jamming (assuming it is detected and reversionary modes can be used). The greater threat is an economic one, where nations could potentially suffer massive impacts in key commercial sectors if GNSS reliant services are denied or interrupted and adequate ‘backup’ or reversionary processes are not developed and regularly practiced. This could be through interruptions to banking, communications, inefficiencies in farming, delays in automated land transport, logistics, etc.

## **How do you think the GNSS positioning technology can take the advantages of other positioning technologies cell phones, Bluetooth and WiFi, etc?**

GNSS technology is complimentary to other positioning technologies and we can expect that they will be increasingly integrated in

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There will be advantages in optimising ‘swarms’ of autonomous vehicles in commercial and military applications to achieve particular outcomes – such as improved efficiency of service, improved logistics or higher probability of military outcomes

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ways that allow the strengths of each solution to be exploited while limitations are mitigated. What will be important will be very reliable and accurate navigation to within centimetres, 24/7, with no shielding effects and at modest cost. This will likely require a variety of globally accessible and low-cost terrestrial high accuracy navigation systems as well as many constellations of GNSS, integrated with high resolution imaging sensors. The more challenging scenarios for the future are likely to be in technology areas such as e-medicine where high accuracy positioning will be essential.

The military, and the development of next generation precision guided munitions in particular, are another area where we are seeing GNSS increasingly augmented with other sensors and positioning technologies. For example, including laser, infra-red, contrast tracking or radar sensors to provide additional guidance for munitions in the terminal phase.

## **What influences you envisage in satellite navigation in the near future given the advancements in the field of AI, Autonomous Vehicles, etc.?**

Arguably, satellite navigation is just one enabler for autonomous vehicles. What is also required are additional ways for the vehicle to sense its surroundings – either through optical, infrared, radar or other imaging means – and then to use the GNSS information, sensor data and an understanding of the environment and rules governing how it should be traversed to determine the optimum way to proceed. Thus high reliability GNSS (including technologies



such as multi-GNSS receivers, anti-spoofing capability and SBAS) would assist, but would not negate the need for integrated sensors and high computing capacity to achieve optimum outcomes in Autonomous Vehicles.

This mutual reliance between GNSS and other technologies (including mobile technologies and 5G) to deliver overall spatial awareness, information and communications services will grow over time, but become less obvious to the users who are just interested in the service being provided.

At a more macro level, there will be advantages in optimising 'swarms' of autonomous vehicles in commercial and military applications to achieve particular outcomes – such as improved efficiency of service, improved logistics or higher probability of military outcomes. This will necessitate the amassing of positional (primarily GNSS) data and then optimisation of the data using Artificial Intelligence systems. An example of this is any future air management system for autonomous air vehicles over built up areas, where there could be several thousand flying in congested areas, all needing to be sent on optimum paths with minimal delays, with adequate safety and with least use of the electrical charge in their batteries. ▷

# “Autonomous navigation will remain the key trend in next two decades”

***says Dr. Capt. Hesham M. Helal, Secretary General Of International Association of Institutes of Navigation (IAIN), President of Arab Institute of Navigation (AIN) and Dean of Maritime Postgraduate Studies Institute, Arab Academy for Science, Technology & Maritime Transport (AASTMT)***

**Would you like to tell us about the International Association of Institute of Navigation (IAIN) such as its objectives and recent activities?**

## History of the IAIN

1957 - A series of conferences sponsored by the British, French and German Institutes of Navigation (ION) were held at three year intervals to discuss technical navigation issues.

1964 - The International Maritime (then Consultative) Organization (IMCO) accepted a report prepared by the three IONs on Traffic Regulation in the Dover Strait.

1968 - A further report on the Separation of Traffic at Sea was accepted by the Maritime Safety Committee of IMCO.

1975 - The Presidents of the Institutes of Australia, France, Germany, Italy, Japan, the United Kingdom and the United States agreed to the formal declaration of an International Association of Institutes of Navigation (IAIN).

1976 - IAIN was granted consultative status to the International Maritime Organization (IMO). Since then many proposals and technical papers on different subjects have been put forward to IMO on such matters as traffic separation, collision regulations and accuracy standards for navigation.

1985 - IAIN was given consultative status to the International Civil Aviation Organization (ICAO). Recommendations on the use of a worldwide geodetic system were made by IAIN.

The aim of the IAIN is to foster human activities at sea, in the air, in space and on land, and who may benefit from the development of the science and practice of navigation and related information techniques.

Some of the objectives of the IAIN are

- Fostering cooperation and assistance between members.
- Establishing technical committees or working groups to study specific problems, and producing appropriate recommendations and standards.
- Organizing Congresses and Seminars relevant to its work.
- Collecting and evaluating information about the activities of its Members, as well as encouraging, supporting and making known recent developments in a periodical.
- Providing assistance to organizations requesting help either technical or organizational, or in training.
- Maintaining liaison with relevant intergovernmental and other organizations and offering specialized advice where appropriate.

### **How do you perceive the evolution and the impact of evolving navigation technologies?**

The current navigation technologies nowadays mean e-navigation, which is used onboard manned ships and of course it is the base for autonomous and unmanned shipping. Parts of shipping are already automated and no doubt that fully autonomous shipping is coming. Even though, there is a lack of investment in innovation and shipping companies are not yet fully engaging, further slowing down the process.

### **In your perspective, what is the general scenario of navigation technology in terms of status, trends and challenges?**

Indeed, the development of navigation technology must be elaborated to avoid unnecessary accidents on one hand, and on the other shipping should preserve the freedom to generate new ideas and implement new applications. Hence, the challenge lies in balancing these competing requirements: a regulatory framework, which protects stakeholders, countries' interests, and current and future seafarers' rights on one side, with the freedom to develop and exploit new technologies on the other.

### **As you also represent Arab Institute of Navigation (AIN), are there any specific trends and challenges you observe in navigation in Arab region?**

In fact, the trends in the Arab countries at the present time, from my point of view, lie in the interest in maritime education and training in line with the use of modern technology in electronic navigation systems.

### **This is a world of multi-GNSS systems. What advantages do you see about this scenario?**

The main advantages of the multi-GNSS systems are:

- Accurate and more robust parameter solutions could be obtained.
- Faster implementations of PPP and PPP-RTK due to the shorter convergence times and RTK to the ability to use longer base lengths for which instant ambiguity resolution is still possible.
- Better tracking performance

also improves positioning in adverse environments.

- Offering opportunities for more precise ionospheric modelling, which in its turn is beneficial again for, for instance, long baseline RTK.

### **What influences you envisage in satellite navigation in the near future given the advancements in the field of AI, Autonomous Vehicles, etc.?**

I expect, of course, that the trend will be for the spread of autonomous ships significantly during the next fifteen to twenty years, and thus a change in the nature of maritime profession and specializations on board ships and the emergence of modern education methods based on artificial intelligence and its applications onboard ships

### **As you have been extensively engaged in maritime education, would you like to highlight the key challenges in maritime education?**

In fact, there are many challenges facing maritime education and training, as the seafarer is accustomed to traditional education, and this includes lecturers and students. Therefore, it has become necessary to change concepts first to accept modern methods that are in line with the developments of modern technology. And then reduce the study materials for marine sciences and increase the educational share related to modern sciences such as artificial intelligence and computer science to keep pace with modern types of ships, especially Autonomous ships. ▽

# “Navigation as a skill will be confined to leisure sailors”

*says John Johnson-Allen MA FRHistS FRIN MIOs, Chairman, Institute of Seamanship in an interview with Coordinates magazine while reflecting on the evolving scenario in marine navigation*



**John Johnson-Allen**  
*MA FRHistS FRIN MIOs*

Joining BP Tanker Co. as an apprentice at the age of 16, he left as a Second Officer.

He maintained his interest in the sea as a Royal Yachting Association lecturer for 22 years, teaching evening classes and also two years at Lowestoft College teaching Iranian ex-commandos technical English and seamanship. He sailed on the East Coast and from the Somme to the Scheldt for over 20 years, and, latterly, on the Norfolk Broads for 10 years. On retirement he took a Masters at Greenwich University in Maritime History, and has written five books and numerous articles for various maritime journals. He is a Liveryman of the Honourable Company of Master Mariners.

**What is purpose of the Institute of Seamanship? Would you like to share some of the recent activities and the achievements of the institute?**

The purpose of the Institute of Seamanship is to promote seamanship through education and training as a practical skill requiring knowledge, experience and common sense. The Institute encourages professional and amateur seamen to extend their seamanship skills and adopt sound seamanship practices. It advances the knowledge and practice of seamanship and supports research into seamanship, past present and future. Recently we have successfully changed the format of our journal from a printed edition to an online edition for members and have created a bursary for supporting young people in acquiring practical seamanship skills through on-board training on a sailing vessel.

**You have extensive experience in marine navigation even of pre-GPS days. Would you like to share some of your experiences and the challenges of those days?**

When I joined British Merchant Navy as a navigating apprentice in 1961 navigation was still little changed from the days of sail. The only electronic help was from the Decca Navigator hyperbolic radio chains, which were used for coastal navigation. All the deep sea navigation was by through the traditional use of astro navigation, using sextants, books of navigation tables and tables of astronomical ephemeris, working out the positions by hand in a sight book. The pattern of sights for a 24-hour period was a star sight at dawn using the Marcq

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The role of the marine navigator as a distinct profession will change as ships are remotely controlled. The role of the navigator will be confined to be shore based, providing and operating the software to navigate the ships of the future. Navigation as a skill will be confined to leisure sailors.

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St Hilaire formula, followed by obtaining a longitude in mid-morning using the sun and the longitude by chronometer formula, transferring the position line thus obtained to the latitude obtained from the noon sun sight which created the noon position from which the days run could be calculated. A further star sight was taken at dusk.

All this, of course, depended on the weather. If overcast, no sights could be taken and positions were estimated by using Dead Reckoning, using the ship's course and speed to give an approximate position.

**How do you see the impact of evolving navigation technologies in marine navigation? Do you think that the advent of satellite-based navigation has redefined the marine navigation?**

The advent of satellite-based navigation transformed the position finding system of navigating a ship completely. In the days before, a position obtained by Astro navigation was accurate to probably a mile, sometimes a little less. Positions now, of course, are almost exact and unless clearly inaccurate, are accepted probably without question.

**Do you think that with the increasing dependence on GNSS, how do you perceive the threats like interference, jamming and spoofing on marine navigation?**

The threats arising from the almost complete dependence on satellite navigation are very real and with the souring of relations between the West and Russia, and the growing influence of China, the use of hacking

techniques will, I believe become more sophisticated and a weapon more widely used. This might result in a return to astro navigation being taught as a backup. That cannot be hacked or interfered with.

**You have taught coastal navigation to yachtsmen in pre-GPS days. What key challenges you perceive in the education of marine navigation?**

The navigation I taught yachtsmen was for coastal navigation, using the traditional methods, using hand held magnetic compasses to obtain bearings, converted to true bearings, coupled with knowledge of tides to compute the tidal effect, and so working out a course to steer. This was taught in a classroom.

In my view the key challenge in the education of marine navigation is the preponderance of shore-based training. The sea time for navigating cadets in the British Merchant Navy is currently 12 months, out of a four-year training course. Whilst simulators are invaluable, they cannot compete with real shipboard experience in real sea time.

**Would you like to convey any message to marine navigators?**

Looking into the future, with the potential for unmanned ships looming, the role of the marine navigator as a distinct profession will change as ships are remotely controlled. The role of the navigator will be confined to be shore based, providing and operating the software to navigate the ships of the future. Navigation as a skill will be confined to leisure sailors. ▽

# “I prefer users and professionals with natural intelligence over those with artificial intelligence”

*says Ing. Jiří Svatoň while discussing the status, trends and challenges of navigation technologies*



**Ing. Jiří Svatoň, Ph.D.**, obtained his Msc. in wireless communications (2013) and Ph.D. in radio electronics (2020) at Czech Technical University in Prague (CTU). He was a postdoc for a short time at EPFL Lausanne (2021) and is currently a scientist at CTU. Jiří also teaches and leads students in signals and systems and radio-positioning. His professional interest is the effective processing of GNSS and general radio-positioning signals and their receivers' construction. He was a Czech Institute of Navigation (CzIN) president from 2016 - 2020.

**Would you like to tell us about the Český Institut Navigace (CZIN) such as its objectives, recent activities and achievements?**

Český institut navigace (original name in Czech language, CzIN in abbreviation. Czech Institute of Navigation in English) is a non-profit professional society for those interested in navigation in the Czech (and Slovak, former states of Czechoslovakia) republic. The CzIN was founded in 1994 (a few years after the Velvet revolution). Nevertheless, the history of a group of founders led by prof. František Vejražka goes back to the turn of the 60' and 70' to means for radars, aircraft radio-positioning, and satellite navigation. Therefore, most of the CzIN members and institute focus historically steams from the field of radio engineering.

CzIN is still one of the small-sized institutes (<100 members). However, Institute is still successful in recruiting young members. I want to remind two milestones in the last decade. Institute changed its abbreviation from the original CIN to the current CzIN (in favor of the China Institute of Navigation). And CzIN was an organizer of the World Congress of the International Association of Institutes of Navigation in Prague in 2015.

**What according to you, is the general scenario of navigation technology in terms of status, trends, and challenges?**

I depict the current era as a reconciliation with the physical limits of traditional GNSS in the users' community and failure to fulfill a dream of a world covering “backup to GPS” in the professional community. GNSS is a “high bar” that is heavy to be overcome on the level of accuracy, availability, continuity, Therefore, some applications like indoor are on their own path. However, both communities look up to a “new deal” in the sense of new GNSS services like high accuracy and authentication and outlook to next-gen positioning, eventually new systems on LEO.

**Are these trends anyway different in Europe in general or in the Czech Republic in particular? What are major emerging application areas of these technologies?**

One decade ago, I admired the dynamic start of BeiDou 2, and my passion was the new S-band signal reception of Indian IRNSS. I

envied the Asia region's revolutionary dynamic in the GNSS field. The development in Europe was more evolutionary. However, the dynamic is comparable now. We are at the same starting point in this era. Europe is ready to be the same innovative leader with the experiences with our Galileo. The Czech Republic's specific, as a country in the center of Europe, is its lack of sea and navy. Therefore we suffer a little from a lack of this traditionally vital part of the navigation community. Nevertheless, we even verified the navigational specifics of the river cruise. And our strong automotive industry is on the other side with its requirements for GNSS products integration and testing. Prague is also the headquarters of Galileo, the EUSPA agency respective. Therefore, a substantial rise of activity is currently in the Czech space industry, with requests for space navigation and communication.

### **This is a world of multi-GNSS systems. What advantages do you see about this scenario?**

In a few words – by high confidence for GNSS accuracy, availability, ... . The user's experience is incomparable to the state before, even for non-professionals. However, in backflash to one of the previous questions, the physical limits and problems of current GNSS are the same.

### **What influences you envisage in satellite navigation in the near future given the advancements in the field of AI, Autonomous Vehicles, etc.?**

As a GNSS professional, I prefer users and professionals with natural intelligence over those with artificial intelligence. Nevertheless, AI is a truly helpful tool for partial tasks. It facilitates scientific research in many ways. Some new challenges are numerically demanding and are not practically solvable without AI. AI certainly does not replace a GNSS professional; it generates new interesting ones. In the sense of autonomous driving, all radio navigation means and inertial sensors must be fused with new progressive means like visual (camera and lidar) perception. And AI can be used like the common alphabet for both worlds. It is a further opportunity to teach and apply something new for traditional GNSS.

### **How do you think the GNSS positioning technology can take the advantages of other positioning technologies cell phones, Bluetooth and WiFi, etc?**

In this area, we periodically study for years. We always hoped for any kind of wide-area usable "GPS backup" by Signal of Opportunity (SoP). Signals of digital wide-band communication and broadcast services promise superior ranging (Time of Flight) parameters. But the snag is in the infrastructure, optimizing its

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I prefer users and professionals with natural intelligence over those with artificial intelligence. Nevertheless, AI is a truly helpful tool for partial tasks. It facilitates scientific research in many ways. Some new challenges are numerically demanding and are not practically solvable without AI

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geometry for data transmission, not for positioning. The terrestrial propagation is a complication too. However, technologically more accessible (3G/4G) cell ID or (WiFi) fingerprinting-oriented positioning aiding services have found a way to our mobile phones in the meantime. We also should remind one reasonable solution in the field of area-limited indoor positioning. It is ultra-wideband positioning. My personal wish in the future is a ranging, not only fingerprinting-based system. It can be like Locata or NextNav, but wider area available and affordable.

### **Do you think that with the increasing dependence on GNSS, how do you perceive the threats like interference, jamming and spoofing?**

We are faced with critical problems with jamming even nowadays. The jamming of the tolling system and close to airport areas were also identified in our country. Do we have better reasons to search for next-gen GNSS than the mixture of jamming, spoofing, requirements for autonomous mobility, and safety of live services?

### **As you are engaged in GNSS education, would you mention the ongoing research trends the key challenges in education?**

The crucial part of our lessons is still the same solid basics of conventional approaches in radio-positioning, signals, systems, signal processing, and PVT computation. And on this basis, we start to teach advances like vector and in-frequency domain tracking and acquisition, direct and distributed positioning. We search for convergence of all these stages. We also pay attention to common theory with communication signals because next-gen navigation and SoP signals will be based on the same principles. Soon will be necessary to start teaching algorithms for new authentication and high-accuracy services, mainly for Galileo in our region. There are many challenges in research and education. ▴

# DEM delivers better approximate AHD heights on public record

This paper outlines how a state-wide Digital Elevation Model (DEM) has been used to deliver better approximate Australian Height Datum (AHD) heights, yielding a homogeneous dataset of known provenance and verifiable quality across New South Wales (NSW), Australia.



**Jonathon Smith**  
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**Dr Volker Janssen**  
Senior Technical Surveyor in the Geodetic Operations team at DCS Spatial Services, NSW Department of Customer Service, in Bathurst, Australia

The Survey Control Information Management System (SCIMS) is the state’s database containing more than 250,000 survey marks on public record across New South Wales (NSW). It was recently updated with Australian Height Datum (AHD; Roelse et al., 1971; Janssen and McElroy, 2021a) heights sourced from a state-wide Digital Elevation Model (DEM) for more than 127,000 survey marks with existing Class U (i.e. approximate) or null AHD height values. Almost 19,000 marks received an AHD height for the first time, while 100 gross AHD height errors in SCIMS were identified and corrected.

The updated AHD heights are classified as Class U and displayed in SCIMS to the nearest metre. They provide important benefits for industry such as enabling the calculation and reporting through SCIMS of the Combined Scale Factor (CSF) and supporting datum modernisation to further improve user access to survey information.

DCS Spatial Services, a unit of the NSW Department of Customer Service (DCS), provides a state-wide DEM with a vertical uncertainty of  $\pm 0.9$  m at the 95% confidence level (CL) and a horizontal grid density of 5 m. While it has been available through the Intergovernmental Committee on Surveying and Mapping’s (ICSM’s) Elevation Information System (ELVIS; ICSM, 2022) as 2 km by 2 km data tiles for some time, the DEM can now be queried directly through a publicly accessible Application

Programming Interface (API) to return an AHD height at a specified location.

This paper outlines how this new interface has been used to query the coordinates of all marks in SCIMS to retrieve AHD heights from the elevation model, assess the accuracy of these AHD heights and update SCIMS with DEM-sourced heights for survey marks with existing Class U or null AHD height values to yield a homogeneous dataset of known provenance and verifiable quality across NSW.

## Background

As surveyors and spatial professionals, we know that height and elevation data is crucial for a vast number of applications and that the survey control we provide underpins this data. DCS Spatial Services delivers various imagery and elevation products as part of its ongoing custodial responsibilities regarding the NSW Foundation Spatial Data Framework.

Clarifying the terminology used, a Digital Elevation Model (DEM) represents the bare-earth surface void of all natural and built features, while a Digital Surface Model (DSM) captures both the natural and built/artificial features of the environment (i.e. including the top of vegetation and buildings).

Accurate and reliable orthorectified aerial imagery and high-resolution

elevation data are critical to effective planning, decision making, change monitoring and risk mitigation across NSW and are utilised by government, industry and the community. Reliable and quality-assured survey control is fundamental to ensuring the integrity of this data, which contributes significantly to economic, social and environmental sustainability in NSW.

As such, the Imagery and Elevation program and project work conducted by the Survey Operations team at DCS Spatial Services supports the following (Powell, 2017):

- Digital Image Acquisition System (DIAS) program, which captures high-resolution 50 cm Ground Sample Distance (GSD) aerial imagery state-wide.
- Digital Town Imagery Capture (DTIC) program, which captures high-resolution 10 cm GSD aerial imagery over cities, towns and villages throughout NSW.
- Light Detection and Ranging (LiDAR) program, which captures highly accurate elevation data in high-risk areas across NSW.
- Surface Model Enhancement (SME) project (2014-19), which utilised a variety of technology including aerial imagery and LiDAR to create a high-resolution, state-wide DSM.

One of the products provided by DCS Spatial Services is a state-wide DEM with a vertical uncertainty of  $\pm 0.9$  m (95% CL) and a horizontal grid density of 5 m. It was produced by a combination of category 1 LiDAR, category 3 LiDAR, 10 cm ground resolution imagery and 50 cm ground resolution imagery. The DEM can now be queried directly through a new publicly accessible API to return an AHD height at a specified location. An API is essentially a connection between computers or between computer programs, i.e. a software interface offering a service to other pieces of software. This is extremely useful when dealing with large amounts of data and/or machine-to-machine processes.

## AHD height retrieval from the DEM via API

In May 2021, AHD heights from the state-wide DEM were extracted via an in-house developed Python script using a publicly available API called 'public/NSW\_5M\_Elevation', hosted by DCS Spatial Services on the NSW Spatial Information Exchange (SIX) platform (DCS Spatial Services, 2022). To obtain

a sufficiently large dataset for evaluation of the accuracy of the returned data, every survey mark in SCIMS (including witness marks, destroyed marks and interstate marks along the borders) was submitted to the API for a height retrieval. This included 301,200 survey marks at the time, using their horizontal Geocentric Datum of Australia 2020 (GDA2020; ICSM, 2021) positions for interrogation.

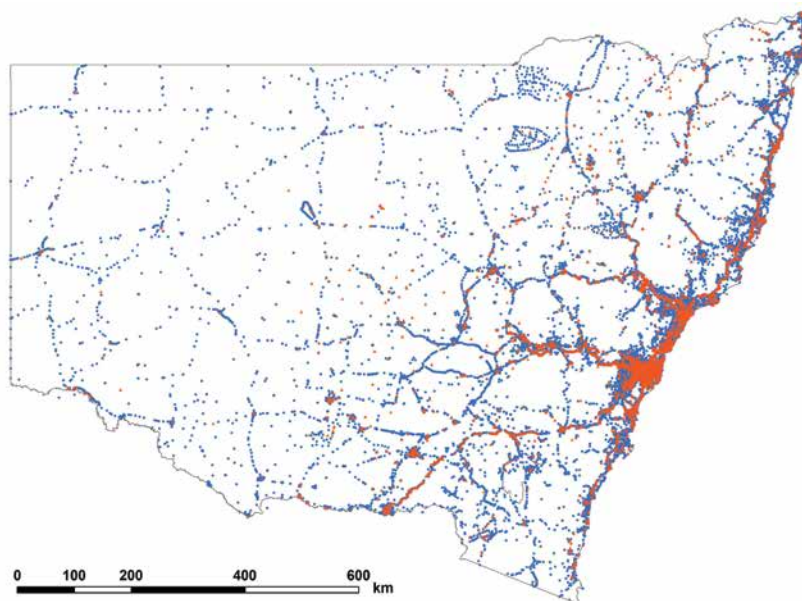


Figure 1: Location of 102,437 SCIMS marks used to assess the quality of the DEM across NSW, with those meeting the  $\pm 0.9$  m threshold indicated in blue.

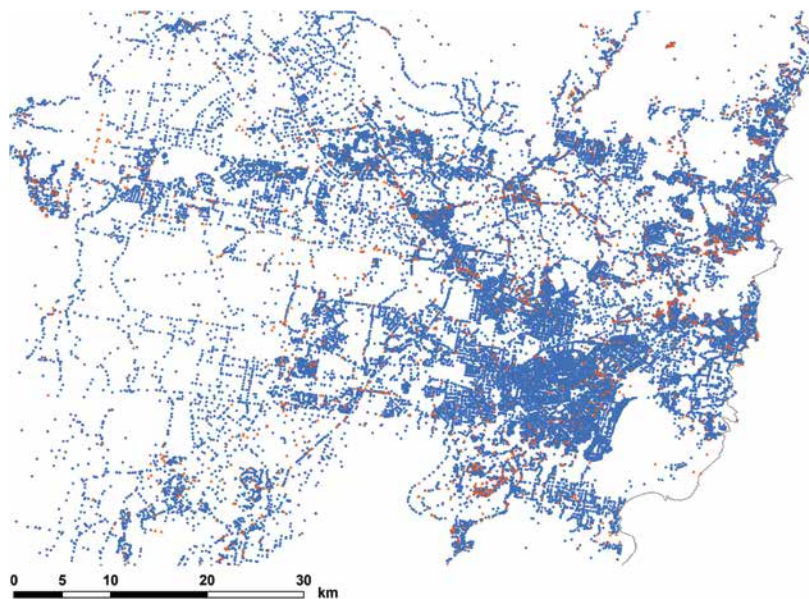


Figure 2: Location of SCIMS marks used to assess the quality of the DEM across Greater Sydney, with those meeting the  $\pm 0.9$  m threshold indicated in blue.



Due to the enormous number of queries required, and to prevent overload of the server, the submission data was separated into sets of 50 marks for asynchronous retrieval, with a wait timer introduced between sets. Retrieval was an iterative process, as failure rates for the server's identify function were as high as 50% of the submitted set at times (likely caused in part by inferior internet connections while working from home during the COVID-19 pandemic). Failed retrievals were re-added to the submission set for the next iteration and automatically resubmitted until completion. This process took approximately 8 days of continuous processor time.

We found that 872 of the submitted positions returned no data, with 46 of these located on Lord Howe Island (which is not covered by this DEM nor true AHD). The remainder (apart from a few anomalies) were located along the Queensland, South Australian and Victorian borders. Closer inspection revealed that all these locations were outside the extent of the DEM (too far into the neighbouring states), noting that SCIMS includes several interstate survey marks close to the NSW border and that the Australian Capital Territory is entirely covered by the DEM.

## Quality assessment of the DEM across NSW

The lower-accuracy AHD heights retrieved from the DEM were compared to existing high-accuracy AHD heights of survey marks on public record in SCIMS that satisfied the following criteria:

- Established (Class D or better) horizontal GDA2020 coordinates.
- Accurate (Class B/LD or better) AHD height.
- Mark at or near ground level (no towers, fence posts, pillars, cairns, reference trees etc.).

The prerequisite for established horizontal coordinates at each survey mark is equally as important as an accurate AHD height, as local terrain undulations can quickly alter the height returned from the DEM.

Similarly, selecting marks that are not on the natural surface (above or below ground) renders the comparison invalid. In this instance, 34% of the survey marks in SCIMS (102,437 of 300,328 that returned heights) met the required criteria to be included in the comparison.

Figure 1 illustrates the distribution of these survey marks across the state, while Figure 2 shows those in Greater Sydney. The threshold for a successful comparison was set at  $\pm 0.9$  m, which is the quoted vertical uncertainty of the DEM (95% CL). It was found that the calculated height differences between the DEM and SCIMS were within this threshold for 95,866 survey marks, i.e. 93.6% of the comparison set (indicated in blue in Figures 1 & 2).

The obtained pass rate of 93.6% is slightly lower than the quoted vertical uncertainty of the model (95%), which can be attributed to two main reasons:

- The comparison did not consider the vertical position of the survey mark above or below ground level (typically up to 0.2 m in either direction for an appropriately placed mark). This was deemed unnecessary as the result was fit for purpose and this information was only available for 3.7% of the comparison set.
- The horizontal density of the DEM is 5 m, so AHD heights at survey marks located on undulating terrain may show some discrepancy, depending on the position of the mark relative to the sample points of the model (which are used to interpolate the height at the specified position).

For at least the last five years, DCS Spatial Services has recorded the mark-to-ground-level offset at each survey mark occupied or inspected as part of normal field operations. To examine the effect of including such metadata, the analysis was repeated for those 3,849 survey marks in the comparison set with available mark-to-ground-level information. This smaller sample exhibited a pass rate of 97.1% when the mark-to-ground-level correction was applied and a 96.2% pass rate when it was ignored.

Together with the earlier analysis, this result was deemed fit for purpose, confirming the stated DEM uncertainty.

It is worth noting that 960 (14.6%) of the 6,571 marks that failed to meet the  $\pm 0.9$  m threshold (indicated in orange in Figures 1 & 2) are located within 20 m of the centreline of a major highway or motorway. This can be explained by rapid changes in topography often occurring along the cross-section of the road corridor, including embankments and cuttings. Furthermore, some of these roads are extremely steep, such as the Great Western Highway leading out of Sydney between Penrith and Glenbrook. Finally, 1,082 (16.5%) of all the marks failing to meet the threshold are specified as 'destroyed' in SCIMS, indicating that their AHD height may relate to a time prior to road or other construction earthworks altering the topography.

## Comparison to an independent, national DEM

Following initial height retrieval, we noted that the DEM returned an AHD height that was significantly different ( $> 20$  m, e.g. a typical contour) from the value in SCIMS for 787 survey marks with an existing Class U AHD height. This was investigated by querying Geoscience Australia's national 1-second Shuttle Radar Topography Mission (SRTM) DEM for the entire dataset via another API (GA, 2022). We then compared the two DEMs to each other and to SCIMS. Wherever the NSW DEM value differed from SCIMS by more than 10 m (in 4,690 cases), the 3-way comparison was recorded.

Any AHD height difference exceeding 20 m between the two DEMs was then manually investigated, resulting in 45 of 108 marks (41.7%) to be identified for exclusion from the following SCIMS update. All these excluded marks were located where an open pit mine had subsequently been created. Any other large differences between the two DEMs were a result of the coarser resolution of the SRTM DEM (1 arcsecond

equates to approximately 30 m), e.g. for Trigonometric Station (TS) pillars located on the side of a cliff, the SRTM DEM sometimes returned the height partway down the cliff. Most remaining large differences between the NSW DEM and SCIMS appeared to be the result of transcription errors in the SCIMS height (e.g. 1,000 m instead of 100 m) or rounding to the nearest contour when the heights were initially entered into SCIMS

## Implementation of DEM-sourced AHD heights in SCIMS

Reliable and quality-assured survey control is fundamental to ensure the integrity of the imagery and elevation products delivered by DCS Spatial Services. However, this connection can work both ways as these products can then be used to improve survey control information on public record in SCIMS. In this case, we updated SCIMS with suitable DEM-sourced AHD heights (at Class U) to yield a homogeneous dataset of known provenance and verifiable quality across NSW. This essentially improved approximate AHD height values in SCIMS that were initially obtained from the nearest contour on 1:25,000, 1:50,000 and 1:100,000 topographic maps to DEM-sourced values with sub-metre uncertainty. Displaying these AHD values to the nearest metre in SCIMS (Class U resolution) fits well with their 0.9 m uncertainty.

Noting the uncertainty of the state-wide DEM, survey marks were only selected for AHD height update if their existing AHD height in SCIMS was null or Class U. Furthermore, it is important to consider that SCIMS holds records for a wide range of different monument types. A location descriptor also indicates whether the mark was placed in the ground or on a structure. As such, further filtering was applied to limit the height update to only those marks that are likely to be at (or near) ground level.

Consequently, several monument types (Table 1) and mark location descriptions

**Table 1: Monument types excluded from the SCIMS update.**

AERIAL	FIRE TOWER	OBELISK	RESERVOIR
BEACON LIGHT	LIGHTHOUSE	RADAR TOWER	SPIRE
CHIMNEY	LIGHTNING ROD	RADIO MAST	TOWER
FLAGSTAFF	MAST	RADIO TOWER	WIND VANE

**Table 2: Mark locations excluded from the SCIMS update.**

BUILDING OR STRUCTURE
SILO
OTHER STRUCTURE

(Table 2) were excluded from the update. As an additional precaution, any TS whose name included the word 'TOWER' was also excluded from the update.

It should be noted that RESERVOIR OR TANK should also have been included on the list of mark locations to be excluded in Table 2 but was unfortunately missed. This resulted in 30 trig stations located on reservoirs incorrectly receiving a height at ground level. These will be revisited and corrected during the upcoming 6-monthly GDA2020 SCIMS refresh.

Once the update set had been filtered in this way and before the SCIMS update was executed, a final test was performed to check for trends. In general, we found that the data was normally distributed and 99.3% of the AHD heights included in the

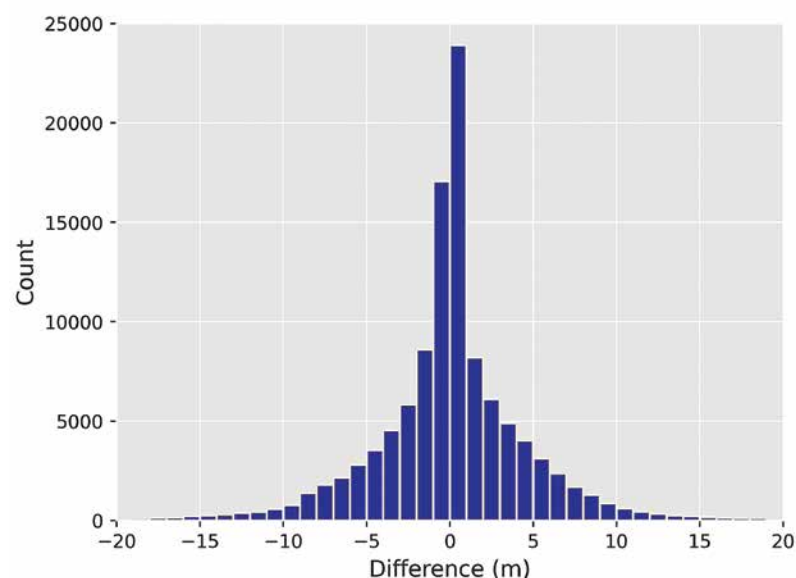
**Table 4: Descriptive statistics of the difference between DEM-sourced and existing AHD height in SCIMS for the update dataset (723 outliers exceeding  $\pm 20$  m were excluded from the calculation of the standard deviation).**

Minimum (m)	-964.725
Maximum (m)	1022.783
Mean (m)	0.185
Median (m)	0.013
Std Dev (m)	4.228

update were within 20 m of their existing SCIMS values (Figure 3). A 20 m error in height corresponds to approximately a 3 parts per million (ppm) error in the reduction of ground distances to the ellipsoid between two marks, which was deemed acceptable and fit for purpose.

As previously mentioned, height differences larger than 20 m were generally attributed to either of the following two reasons:

- Transcription or rounding errors in the existing SCIMS value (corrected by the update).
- Marks located where an open



**Figure 3: Histogram of the difference between DEM-sourced AHD height and existing AHD height in SCIMS (723 outliers exceeding  $\pm 20$  m not shown).**

pit mine had subsequently been created (excluded from the update with mark status updated in SCIMS as 'destroyed').

## SCIMS update results

This SCIMS update delivered DEM-sourced AHD heights at Class U for 127,154 survey marks, of which 18,854 marks (14.8%) were assigned an AHD height for the first time (Figure 4). Putting this large number into perspective, this means that 42.3% of the survey marks that returned a DEM-sourced AHD height during the initial retrieval were updated during this process. This is a huge improvement in the access to reliable, approximate AHD heights of known quality in SCIMS, ensuring that nearly every survey mark in NSW has an AHD value of 0.9 m uncertainty or better.

When inspecting Figure 4, it is worth noting the near-perfect straight line of AHD height updates to survey marks running from north-west to south-east through the centre of the state. Despite appearing to be an artefact, this is a series of marks located along a gas pipeline easement, which connects to the main distribution network on the east coast.

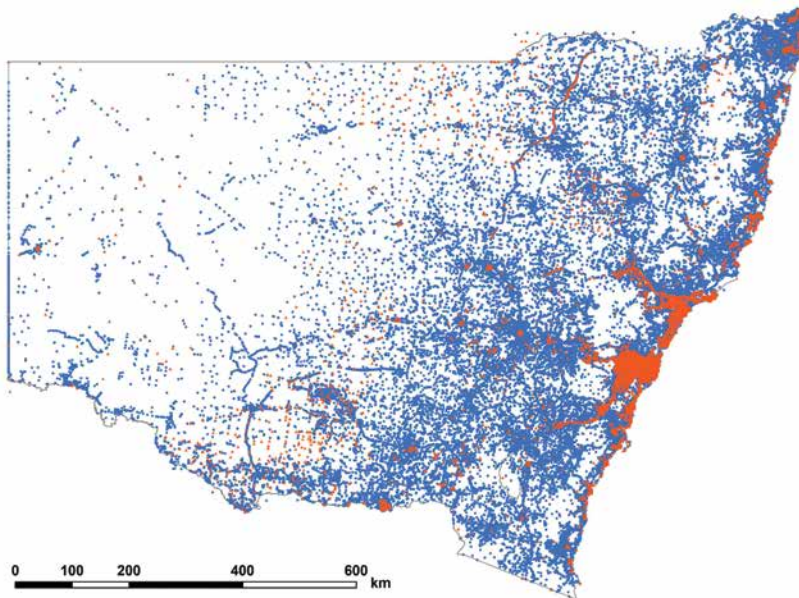


Figure 4: Location of survey marks included in the May 2021 SCIMS update, indicating which marks received an AHD height for the first time in orange.

Table 4 summarises descriptive statistics related to the update dataset, showing the minimum, maximum, mean and median differences between the new and existing AHD height in SCIMS along with the resulting standard deviation. The existence of large outliers, as previously discussed, is confirmed by the difference between the mean and median values.

## Benefits

These updated AHD heights provide several important benefits such as enabling better calculation and reporting of the Combined Scale Factor (CSF) with confidence at virtually all survey marks (99.98%) in NSW through SCIMS. CSFs are now typically up to 1.5 ppm better because heights have been improved from 10-metre to sub-metre accuracy. The DEM-sourced AHD heights also support datum modernisation efforts through the ongoing readjustment of legacy terrestrial data hosted by DCS Spatial Services for inclusion in the growing GDA2020 state adjustment by facilitating the rigorous reduction of terrestrially measured distances to the ellipsoid. This translates into more survey marks in SCIMS being assigned a Positional Uncertainty (PU), directly benefitting the profession and our customers.

Furthermore, this process was able to identify and correct 101 extremely large (100-1,000 m) AHD height errors (Class U) in SCIMS. It follows that retrieved DEM-sourced heights can now also be used to identify gross errors on SCIMS marks with existing accurate (Class B/LD or better) AHD heights. This further contributes to our 'Saving AHD' efforts, which aim to ensure that users have continued and easy access to reliable physical heights and their uncertainties across NSW (Janssen and McElroy, 2021b).

Finally, with SCIMS now holding AHD heights of known quality state-wide, ellipsoidal height was derived by applying AUSGeoid2020 (e.g. Brown et al., 2018; Janssen and Watson, 2018; Featherstone et al., 2019) at all applicable survey marks with existing null ellipsoidal height values in SCIMS. During the 6-monthly GDA2020 SCIMS refresh in November 2021, this provided ellipsoidal heights for 267,581 survey marks for the first time, ensuring that virtually all marks in SCIMS now also have an ellipsoidal height. Publishing these values allows surveyors and other users to easily verify that they have set their height datum and/or applied AUSGeoid2020 correctly during both field operations and office processing and reductions.

## Conclusion



DCS Spatial Services provides a state-wide DEM to the public with a vertical uncertainty of  $\pm 0.9$  m (95% CL) and a horizontal grid density of 5 m. This DEM can now be queried directly through a new publicly available API to return an AHD height at a specified location. This paper has described how the new API was used to retrieve AHD heights from the elevation model for about 300,000 survey marks in SCIMS and assess the accuracy of these DEM-sourced AHD heights, thereby confirming the stated DEM uncertainty.

We have outlined how SCIMS was updated with DEM-sourced heights (at Class U) for 127,154 survey marks with

existing Class U or null AHD height values to deliver a homogeneous dataset of known provenance and verifiable quality across NSW. This allowed 18,854 survey marks to be assigned an AHD height for the first time, while many gross AHD height errors in SCIMS were identified and corrected.

The updated AHD heights provide important benefits for industry such as enabling the calculation and reporting through SCIMS of the CSF with confidence across all of NSW and supporting the readjustment of legacy terrestrial data in the growing GDA2020 state adjustment. Lastly, ellipsoidal height values were derived using AUSGeoid2020 and published in SCIMS, allowing 267,581 survey marks to receive an ellipsoidal height for the first time. As a result, virtually all SCIMS marks now have both an AHD height and ellipsoidal height.

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- ## EUSPA taps ESSP for EGNOS service provider role
- The European Geostationary Navigation Overlay Service (EGNOS) is Europe's regional satellite-based augmentation system (SBAS). Used to improve the accuracy and reliability of GNSS positioning information, EGNOS is designed to provide safety of life navigation services to aviation, maritime and land-based users across the EU.
- While the exploitation of EGNOS is the responsibility of the EU Agency for the Space Programme (EUSPA), its services are delivered by the EGNOS service provider under a contract with EUSPA. Recently, EUSPA formally announced that it has signed its new EGNOS service provider contract with European Satellite Services Provider (ESSP)
- The new 10 years contract sees ESSP continuing its role as the EGNOS service provider for the Open Service and Safety of Life Service (SoL) while EUSPA is in the process of taking over the responsibility of EGNOS Data Access Service (EDAS) service provision. During this time, ESSP will be responsible for EGNOS service provision (including EGNOS operations and part of its maintenance).
- ## Expanding and evolving
- With EGNOS constantly expanding and evolving, the new contract will see ESSP performing some new tasks as well. For example, in addition to bolstering EGNOS' use in the aviation sector, the company will look to further develop the service for the maritime, rail and drone sectors. It will also help improve the security of EGNOS V2 through the addition of new functions and by upgrading the system.
- Then there's EGNOS V3, the next generation of EGNOS that will augment Galileo signals. With ESSP set to play a major role in this transition, the company has brought on new partners, including Airbus Defence and Space currently in charge of the development of EGNOS V3.
- Under the new contract, ESSP will work to further expand the EGNOS services in European Neighbourhood Policy South countries (ENP-South) and Ukraine. The company is in the process of setting up new RIMS in Nigeria and Chad, the operation of which will be subcontracted to the Agency for Aerial Navigation Safety in Africa and Madagascar (ASECNA). 

# Generalized orthogonalization an elegant solution to the least squares problem

The generalized orthogonalization does not require the formation of the normal equations and is therefore numerically more stable than the conventional method



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## Abstract

The purpose of this article is to present a resolution method to the users of the least squares method in geodesy and topography and which has considerable advantages. This so-called “Generalized Orthogonalization” method is based on a modified version of the Gram-Schmidt orthogonalization method. It is very concise, numerically stable and allows the solution of simply or overdetermined systems of linear equations. It does not use any matrix inversion algorithm or the successive elimination technique. There is no formation of normal equations.

## Introduction

The least squares method was invented by Legendre to determine the orbital parameters of the planets. It was a little later that Gauss gave this method the foundations and rigorous mathematical formulations.

Since then, many mathematicians have contributed to the development and applications of this method. Let us quote Laplace, Tchebychev, Kalman and Markov. Significant work has also been carried out by geodesists such as Doolittle, Helmert, Tienstra, Meissl and Moritz. In Belgium we must refer to the works of Marchant, Baetslé and Van Den Herrewegen. We have also associated with certain algorithms, the name of those who proposed original methods allowing the numerical resolution associated with

this method as in France Cholesky and Levallois. The other Banachiewicz, Gauss and Jordan are well known.

Today, the computer tools at our disposal require adapted numerical methods. It is agreed that they must meet a certain number of criteria including numerical stability, ease of programming, portability of the software, that is to say its independence to a specific operating system, etc.

The purpose of this paper is to present a method of resolution little known to users of the method of least squares in geodesy and surveying and which nevertheless has considerable advantages. This so-called “Generalized Orthogonalization” method is based on a modified version of the Gram-Schmidt orthogonalization method. It is very concise, numerically stable and allows the solution of simply or overdetermined systems of linear equations. It does not use any matrix inversion algorithm or the successive elimination technique. There is no formation of normal equations. The pioneers of this method are F. Charamza [1] and J. Gazdzicki [2].

We had already published an article on this method in 1986 in the Bulletin Trimestriel de la Société Belge de Photogrammétrie-Télédétection et Cartographie [3] when we worked at the National Geographic Institute of Belgium in the Department of Geodesy from 1983 to 1989. At the time the first mini-computers were appearing and the Department of Geodesy wanted

to free itself from the services of the Computer Processing Center (CTI) which worked in time sharing on IBM servers at high prices charged per millisecond of CPU . To give an example, the processing of the aerial triangulation of Rwanda took 20 minutes and exhausted the annual budget of the CTI. This means that a certain independence was welcome.

Our mission was to develop a least squares adjustment software, including all the numerical refinements of the time, namely the B test method invented by Professor Baarda of Delft [15] for the detection of gross errors but also to produce the indicators internal and external reliability and the adjustment of free networks that call for generalized inversion.

Generalized orthogonalization was systematically used in our software at that time but also for all the software that we developed afterwards and in particular for example the Star Topo TX software from Star Informatic [16].

There was another request to allow the second-order design of a geodetic network where the position of the points is imposed and where one wishes, to achieve a given precision in terms of standard deviation, to calculate the precision to be obtained for each observation. Second-order design is one of the most important orders in geodetic network design. In this design order, the optimal weights of the observations are sought. It also used a very particular matrix product invented by Khatri and Rao [4] used in a development by Gunther Schmitt [5].

At that time, there were very important international developments and contributions related to the numerical solution of the least squares problem. With limited means of digital processing, the imagination was highly solicited. Today the processing of millions of points from the famous point clouds generated by scanners and other sensors no longer surprises many people. On the other hand, students but also professionals in geodesy and surveying no longer seem motivated to dive into these algorithms

with the risk of gradually losing their mastery and therefore becoming dependent on software production companies.

For my part, at that time, as a young graduate Surveyor-Expert with a background in Mathematics, I set myself the rule of professional life not to use any processing method that I could not both demonstrate and therefore program. This allowed me to understand exactly how the results of the observations were delivered and also to optimize the surveying operations and to innovate through the choice of mathematical models. In this respect, the arrival of GPS and then GNSS has been a godsend for many new developments, all of which are nevertheless based on this knowledge base of the least squares method coupled with digital filtering and statistical inference.

The teaching of mathematics, currently in a bad situation, aims, among other things, to discover a certain beauty. Bruno Hourst writes [6] Mathematics can be a source of beauty. We can speak of a “beautiful formula” or of a “beautiful geometric demonstration”. And indeed, when one has felt this beauty, it seems obvious. Geometric demonstrations are “beautiful” by the elegance of the reasoning and the articulation of the demonstration. Some formulas are “beautiful” by their simplicity or their balance.

Semir Zeki [6] explained: “The beauty of a formula can result from simplicity, symmetry, elegance or the expression of an immutable truth. For Plato, the abstract quality of mathematics expressed the pinnacle of beauty”.

Let’s quote Carl Friedrich Gauss again, about mathematics “The enchanting charms of this sublime science are revealed in all their beauty only to those who have the courage to explore it in depth. “[17]

This article is therefore also an invitation to discover the elegance of an algorithm that takes another path to solve the problem of least squares. And indeed in the question of finding the minimum distance

between a point and a line, one can either minimise the distance function to this line or to construct the perpendicular of this point lowered on the line. Generalized orthogonalization is quite another way of solving the least squares problem by producing unbiased linear estimators in overdetermined systems of equations.

## Orthogonalization

### Directions

We will present in this paragraph a reminder of the mathematical notions used in this article. Unless otherwise specified, all vectors will henceforth be column vectors.

When the components are explained, we will write  $[x_1, x_2, \dots, x_n]^T$ . The symbol  $[ ]^T$  indicates that the elements must be written in columns.

$n$ -component vectors over the set  $F$  is addition-stable, if the sum of any two of them is still a vector of the set. Similarly the set is stable for multiplication by the scalars if any vector of the set multiplied by a scalar is still a vector of the set.

### Vector spaces

Any set of  $n$ -component vectors over  $F$  stable for multiplication by scalars and for addition is called a **vector space** .

Thus, if  $X_1, X_2, \dots, X_m$  are  $n$ -component vectors over  $F$ , the set of linear combinations

$$k_1X_1 + k_2X_2 + \dots + k_mX_m \quad \text{with } k_i \text{ in } F \text{ is a vector space over } F.$$

The space  $V_n(F)$  of all  $n$ -component vectors over  $F$  is called the  $n$ -dimensional vector space over  $F$ .

$V_n(R)$  will be the  $n$ -dimensional vector space over  $R$ , where  $R$  is the set of reals.

### Subspaces

A set  $V$  of vectors of  $V_n(F)$  is a subspace

of  $V_n(F)$  if  $V$  is stable for addition and multiplication by scalars. Thus, the  $n$ -dimensional non null vector space is a subspace of  $V_n(F)$ . It is the same with  $V_n(F)$  himself.

### Basis and Size

The dimension of a vector space  $V$  is the minimum number of linearly independent vectors to generate  $V$ . The set of these vectors is called a basis of  $V$ . If these are orthogonal two by two, the so-called orthogonal basis. If their norm is equal to unity, the basis will be said to be orthonormal.

In elementary geometry, the usual space is considered as a 3-dimensional set.

The so-called elementary vectors will be:

$$\begin{aligned} E_1 &= [1, 0, 0]^T \\ E_2 &= [0, 1, 0]^T \\ E_3 &= [0, 0, 1]^T \end{aligned}$$

These constitute an important basis of  $V_3(F)$  what is called the canonical basis of  $V_3(F)$ .

### Gram-Schmidt orthogonalization

In linear algebra, in a pre-Hilbertian space (i.e. a vector space over the field of real numbers or that of complexes, endowed with a scalar product), the Gram-Schmidt process or algorithm is an algorithm for constructing, from a finite free family, an orthonormal basis of the subspace it generates. This method was published by Jørgen Pedersen Gram in 1883 and reformulated by Erhard Schmidt in 1907, but it is already found in works from 1816 by Laplace and by Cauchy in 1836 [7].

Let be  $X_1, X_2, \dots, X_m$  a given base of  $V_n^m(R)$ .

We denote by  $Y_1, Y_2, \dots, Y_m$  the set of two-by-two orthogonal vectors to be determined.

Step 1: We take  $Y_1 = X_1$

Step 2: Then we take  $Y_2 = X_2 + aY_1$

Since  $Y_1$  and  $Y_2$  are orthogonal, we have

$$Y_1 * Y_2 = Y_1 * X_2 + aY_1 * Y_1 = 0$$

$$\text{And } a = -\frac{Y_1 * X_2}{Y_1 * Y_1}$$

$$\text{Thus } Y_2 = X_2 - \frac{Y_1 * X_2}{Y_1 * Y_1} * Y_1$$

Step 3: We take  $Y_3 = X_3 + aY_2 + bY_1$

Since  $Y_1, Y_2$  and  $Y_3$  are pairwise orthogonal

$$\begin{aligned} Y_1 * Y_3 &= Y_1 * X_3 + aY_1 * Y_2 + bY_1 * Y_1 = Y_1 * X_3 + bY_1 * Y_1 = 0 \\ Y_2 * Y_3 &= Y_2 * X_3 + aY_2 * Y_2 + bY_2 * Y_1 = Y_2 * X_3 + aY_2 * Y_2 = 0 \end{aligned}$$

$$a = -\frac{Y_2 * X_3}{Y_2 * Y_2} \quad \text{and} \quad b = -\frac{Y_1 * X_3}{Y_1 * Y_1}$$

$$\text{and} \quad Y_3 = X_3 - \frac{Y_2 * X_3}{Y_2 * Y_2} * Y_2 - \frac{Y_1 * X_3}{Y_1 * Y_1} * Y_1$$

We continue this method until we obtain  $Y_m$

$$Y_m = X_m - \frac{Y_{m-1} * X_m}{Y_{m-1} * Y_{m-1}} * Y_{m-1} - \dots - \frac{Y_1 * X_m}{Y_1 * Y_1} * Y_1$$

Then the unit vectors  $G_i = \frac{Y_i}{\|Y_i\|}$ , ( $i = 1, 2, \dots, m$ ) are pairwise orthogonal and form an orthonormal basis of  $V_n^m(R)$

Note: the symbol  $*$  is the scalar product between two vectors.

Example: Construct using the Gram-Schmidt orthogonalization process an orthogonal basis of  $V_3(R)$  given the following basis:

$$\begin{aligned} X_1 &= [1, 1, 1]^T \\ X_2 &= [1, -2, 1]^T \\ X_3 &= [1, 2, 3]^T \end{aligned}$$

$$\text{Step 1 : } Y_1 = X_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$\text{Step 2 : } Y_2 = X_2 - \frac{Y_1 * X_2}{Y_1 * Y_1} * Y_1 = \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix} - \frac{0}{3} * Y_1 = \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$$

$$\text{Step 3 : } Y_3 = X_3 - \frac{Y_2 * X_3}{Y_2 * Y_2} * Y_2 - \frac{Y_1 * X_3}{Y_1 * Y_1} * Y_1 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} - \frac{0}{6} * Y_2 - \frac{6}{3} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$$

So the vectors

$$G_1 = \frac{Y_1}{\|Y_1\|} = \begin{bmatrix} 1 \\ \sqrt{3} \\ 1 \\ \sqrt{3} \end{bmatrix}; \quad G_2 = \frac{Y_2}{\|Y_2\|} = \begin{bmatrix} 1 \\ \sqrt{6} \\ -2 \\ \sqrt{6} \\ 1 \\ \sqrt{6} \end{bmatrix}; \quad G_3 = \frac{Y_3}{\|Y_3\|} = \begin{bmatrix} -1 \\ \sqrt{2} \\ 0 \\ 1 \\ \sqrt{2} \end{bmatrix}$$

forms an orthonormal basis of  $V_3(R)$ .

Any vector  $G_i$  will be unit vector and any scalar product  $G_i * G_j = 0$

### Orthogonal matrix

A square matrix  $A$  is orthogonal if:  $A^T A = A A^T = I$

That is to say if:  $A^T = A^{-1}$

It is clear that the column vectors (respectively rows) of an orthogonal matrix  $A$  are pairwise orthogonal unit vectors.

Let be  $Y = AX$  a linear application of  $V_n(R)$ . If it is a coordinate transformation from one base  $E$  to another base  $Z$ , the base  $Z$  will be orthogonal if  $A$  is orthogonal.

The matrix  $A$  formed by the vectors  $G_1, G_2$  and  $G_3$  of our previous example is orthogonal.

### Orthogonalization and least squares method

Consider the problem of finding a vector belonging to  $V_m(R)$  such that  $AX = B$  where  $A$  belongs to  $V_n^m(R)$  and  $B$  belongs to  $V_n(R)$  with  $n \geq m$ .

When there are more equations than unknowns, the system of equations is said to be overdetermined. The least squares problem consists in minimizing the Euclidean norm:

$$\min_x \|AX - B\|$$

$A$  belonging to  $V_n^m(R)$  and  $B$  belonging to  $V_n(R)$

We ask for an optimal estimator for the vector  $X$  belonging to  $V_m(R)$ .

The Euclidean norm being invariant with respect to orthogonal transformations, the least squares method benefits from the interesting property of being equivalent to the following problem:

$$\min_x \|(Q^T A)X - (Q^T B)\|$$

by multiplying  $A$  and  $B$  by an orthogonal matrix  $Q^T$ .

Suppose that the orthogonal matrix  $Q^T$  has been calculated in such a way that:

$$Q^T A = \begin{bmatrix} R \\ 0 \end{bmatrix}_{n-m}^m \text{ and } Q^T B = \begin{bmatrix} C \\ D \end{bmatrix}_{n-m}^m$$

The solution of the least squares problem is then obtained by solving the upper triangular system:

$$RX = C$$

Moreover the norm of the vector des residuals  $V = AX - B$

will be equal to the norm of the vector  $D$ .

$$\|AX - B\|^2 = \|Q^T AX - Q^T B\|^2 = \|RX - C\|^2 + D^2$$

Three methods are generally proposed to calculate this orthogonal factorization. These are Gram-Schmidt orthogonalization, Householder reflectors and Givens rotations.

The method presented here is based on Gram-Schmidt orthogonalization. Unfortunately, this one has a rather low numerical stability. To overcome this, an equivalent version called "modified Gram-Schmidt" has been proposed where the normalization of the vectors is done at each step of the transformation [8] and [9].

## Generalized orthogonalization

### Directions

The generalized orthogonalization algorithm is based on the transformation of a matrix  $A$  into a matrix  $W$  of the same structure.

$$A = \begin{matrix} & \begin{matrix} m & r \end{matrix} \\ \begin{matrix} n \\ p \end{matrix} & \begin{bmatrix} A_1 & A_2 \\ A_3 & A_4 \end{bmatrix} \end{matrix} \quad W = \begin{matrix} & \begin{matrix} m & r \end{matrix} \\ \begin{matrix} n \\ p \end{matrix} & \begin{bmatrix} W_1 & W_2 \\ W_3 & W_4 \end{bmatrix} \end{matrix}$$

respecting the following rules:

Rule 1: Orthogonalization in the submatrix  $[A_1 \ A_3]^T$

This submatrix is transformed  $[W_1 \ W_3]^T$  using modified Gram-Schmidt orthogonalization. The normalization, that is to say the division by the norm of the vector, only applies to the matrix  $A_1$ . This has its columns linearly independent.

Rule 2: Orthogonalization in the sub-matrix  $[A_2 \ A_4]^T$

This sub-matrix is transformed by  $[W_2 \ W_4]^T$  applying the sequence of transformations already applied for the sub-matrix  $[A_1 \ A_3]^T$  but without normalization.

The procedure can be described as follows:

Let  $h$  be the number of transformations

$$\begin{aligned} h &= 1, 2, \dots, m \\ \text{For } j &= 1, 2, \dots, n, n+1, \dots, n+p \\ i &= h+1, h+2, \dots, m+r \end{aligned}$$

Do

$$W_{hj}^h = \frac{W_{hj}^{h-1}}{\sqrt{\sum_{k=1}^n W_{hk}^{h-1}{}^2}}$$

$$W_{ij}^h = W_{ij}^{h-1} - W_{hj}^h * \sum_{k=1}^n W_{hk}^h * W_{ij}^{h-1}$$

$W_{hj}^h$  is noted as being the element  $W_{hj}$  obtained after  $h$  transformations. The transformation  $h = 1$  being the initial step,

$$W_{hj}^0 = A_{nj}$$

Thus  $W_1$  becomes the orthonormal transformation of  $A_1$ . We can verify that  $W_1^T W_1 = I$ . The sub-matrices  $A_1$  and  $W_1$  being a function of a non-singular right triangular matrix  $R$  such that

$$A_1 = W_1 R$$

By applying the described procedure, we obtain the following fundamental relations of the generalized orthogonalization:



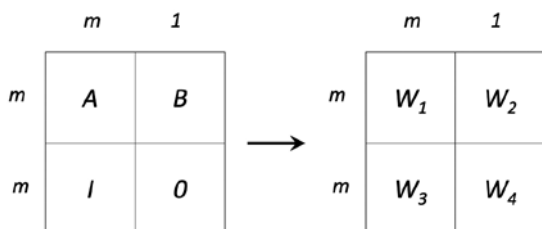
$$\begin{aligned}
 W_1 &= A_1 R^{-1} & (1) \\
 W_3 &= A_3 R^{-1} & (2) \\
 W_2 &= A_2 - W_1 W_1^T A_2 & (3) \\
 W_4 &= A_4 - W_3 W_3^T A_2 & (4)
 \end{aligned}$$

We will show later how to define the matrices  $A_1, A_2, A_3$  and  $A_4$  to obtain in a variety of cases the solution to the problems posed by the resolution of a simply determined or overdetermined system of linear equations.

### System of simply determined linear equations

Either to determine the vector  $X$  in the equation  $AX = B$

The solution is obtained by multiplying both sides of the equation by the inverse matrix  $A^{-1}$ . Either  $A^{-1}AX = A^{-1}B$  and therefore  $X = A^{-1}B$

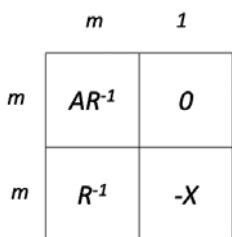


By applying the fundamental relations (1), (2), (3) and (4), we obtain:

$$\begin{aligned}
 W_1 &= AR^{-1} \\
 W_3 &= R^{-1} \\
 W_2 &= B - W_1 W_1^T B \\
 W_4 &= 0 - W_3 W_3^T B
 \end{aligned}$$

like  $W_1 W_1^T = I$ , the matrix  $W_1$  being square we can deduce that  $W_1^T = W_1^{-1}$  and therefore:

$$\begin{aligned}
 W_2 &= 0 \\
 W_4 &= 0 - R^{-1} W_1^T B \\
 W_4 &= -R^{-1} (R A^{-1}) B \\
 W_4 &= -A^{-1} B = -X
 \end{aligned}$$



It is therefore possible to calculate the inverse of a regular matrix using this algorithm.

### System of overdetermined linear equations

Depending on whether the observations are expressed as a function of certain parameters (coordinates) or whether certain conditions are imposed on them, we will have different models of equations, leading however to identical results.

These are the model of observation equations (Gauss-Markov) and the model of condition equations (Gauss-Helmert).

One can be reduced to the other by algebraic elimination of relations between unmeasured quantities, but this is a problem of only theoretical interest [10] and [11].

There are obviously many other models adapted to more complex situations but which are only a category relating to one or other of the models mentioned.

Note also that due to the nature of the observations that are processed in topography and geodesy, certain established functions are not linear (directions/angles and distances). The implementation of the method of least squares requires that these equations be made linear by approximation by replacing them with a series expansion limited to linear terms.

### Observation equations

We will briefly remind the general equations:

Consider the following linear functional model:

$$L + V = AX + A_0 \tag{5}$$

Where

$L$  is the vector of observations (n)

$V$  is the vector of residuals (n)

$A$  is the matrix of the coefficients of the observation equations (nxm)

$X$  is the vector of the parameters to be estimated (m)

$A_0$  is the vector of constant terms (n) integrated in practice into the vector  $L$

The stochastic model containing the probabilistic information of the observations is defined as follows:

- $\sigma^2$  scalar quantity, referred to as the variance of the unit weight
- $Q_{LL}$  is the observation cofactor matrix (nxn)
- $C_{LL} = \sigma^2 Q_{LL}$  is the variance-covariance matrix of the observations
- $P_{LL} = C_{LL}^{-1}$  is the (n x n) matrix of weights

We obtain the following estimators:

For parameters:

$$\tilde{X} = (A^T P A)^{-1} (A^T P L) \tag{6}$$

or by setting  $N = A^T P A$  we have:  $\tilde{X} = N^{-1}(A^T P L)$

For observations:  $\tilde{L} = A\tilde{X} = AN^{-1}A^T P L$  (7)

For residuals:  $\tilde{V} = \tilde{L} - L = A\tilde{X} - L$

For a posteriori variance-covariance matrices:

For parameters:  $C_{\tilde{x}\tilde{x}} = \sigma^2 N^{-1}$  (8)

For observations:  $\tilde{L} = A\tilde{X} = AN^{-1}A^T P L$  (9)

For residuals:  $\tilde{V} = \tilde{L} - L = A\tilde{X} - L$

The a posteriori variance factor of the estimated weight unit:  $\tilde{\sigma}^2 = \frac{\tilde{v}^T P \tilde{v}}{n-m}$  (10)

Let's define the following operator matrix:

	$m$	$1$
$n$	$A$	$L$
$m$	$I$	$O$

After transformation, we obtain by applying (1), (2), (3) and (4):

$$\begin{aligned} W_1 &= AR^{-1} \\ W_3 &= IR^{-1} = R^{-1} \\ W_2 &= L - W_1 W_1^T L \\ W_4 &= -R^{-1} W_1^T L \end{aligned}$$

By explaining these relations, we obtain the following results:

$$\begin{aligned} W_4 &= -R^{-1} W_1^T L \\ W_4 &= -R^{-1} R^T R^{-1} R^T W_1^T L \\ W_4 &= -(R^T W_1^T W_1 R)^{-1} R^T W_1^T L \text{ with } W_1^T W_1 = I \text{ and } W_1 R = A \\ W_4 &= -(A^T A)^{-1} A^T L \end{aligned}$$

$W_4 = -\tilde{X}$  the solution parameter vector

$$\begin{aligned} W_2 &= L - W_1 W_1^T L \\ W_2 &= L - W_1 R R^{-1} W_1^T L \text{ with } R R^{-1} = I \\ W_2 &= L - A(R^{-1} W_1^T L) \\ W_2 &= L - A\tilde{X} \end{aligned}$$

$W_2 = -\tilde{V}$  the residual vector

$$W_1 W_1^T = A(A^T A)^{-1} A^T$$

$W_1 W_1^T = Q_{\tilde{L}\tilde{L}}$  the cofactor matrix of the adjusted observations

$$W_3 W_3^T = R^{-1} R^{-1^T} = (R^T R)^{-1} \quad R = W_1^{-1} A$$

$$W_3 W_3^T = (A^T W_1^{-1^T} W_1^{-1} A)^{-1}$$

$$W_3 W_3^T = (A^T A)^{-1}$$

$W_3 W_3^T = Q_{\tilde{x}\tilde{x}}$  the matrix cofactors of the parameters solution

We thus find after transformation in the matrix operator, the following information:

	$m$	$1$
$n$	$AR^{-1}$	$-V$
$m$	$R^{-1}$	$-X$

To obtain the diagonal elements of the cofactor matrices  $Q_{\tilde{L}\tilde{L}}$  and  $Q_{\tilde{x}\tilde{x}}$  just to sum the squared elements per row.

To introduce, if necessary, an estimation of the a priori variance-covariance matrix, it can be shown that it suffices to divide each row of the coefficients of the matrix  $A$  and of the vector  $L$  by the square root of the variance of the corresponding observation. In the case of a priori correlated variance-covariance matrix, it will be necessary to factorize the inverse matrix, known as the matrix of weights, into two triangular matrices (by Choleski processes [18] or Givens [19] for example) and multiply the configuration matrix and the vector of observations by one of them.

Indeed, the matrix of weights is usually positive definite and can be decomposed into a left ( $G$ ) and ( $D$ ) right triangular matrix .

We obtain therefore:  $P = GD = GG^T = D^T D$

The expression:  $(A^T P A)X = A^T P L$

Becomes:  $(A^T D^T D A)X = A^T D^T D L$

Where we put  $A^* = DA$  and  $L^* = DL$

And we get:  $(A^{*T} A^*)X = A^{*T} L^*$

The operator matrix is therefore presented as follows:

	$m$	$1$
$n$	$A^*$	$L^*$
$m$	$I$	$O$

In a GNSS baseline adjustment software, we had taken as the variance-covariance matrix of the pseudo-observations (the components of the baselines) the variance-covariance matrix resulting from the post-processing software to note that the triangular decomposition did not have an inverse. By superimposing variances estimated as being a function of distance on the main diagonal, we solved the problem.

## Conditional equations

Consider the following linear functional model:

$$UL + U_0 = F \quad (11)$$

With :

$U$  configuration matrix (m, n)

$L$  vector of observations (n)

$U_0$  vector of constant terms (m)

$F$  vector of closures (m)

As the stochastic model relates to the same observations, it will be identical to that described in paragraph 3.3.1

We obtain the following estimators:

$$\text{Correlatives: } K = (UQ_{LL}U^T)^{-1}F \quad (12)$$

$$\text{Observations: } \tilde{L} = L + \tilde{V} \quad (13)$$

$$\text{Residuals: } \tilde{V} = -Q_{LL}U^T(UQ_{LL}U^T)^{-1}F = -Q_{LL}U^TK \quad (14)$$

And for the a posteriori variance-covariance matrices:

$$\text{Correlatives: } C_{\tilde{K}\tilde{K}} = \sigma^2(UQ_{LL}U^T)^{-1} \quad (15)$$

$$\text{Observations: } C_{\tilde{L}\tilde{L}} = \sigma^2(Q_{LL} - Q_{LL}U^T(UQ_{LL}U^T)^{-1}UQ_{LL}) \quad (16)$$

$$\text{Residuals: } C_{\tilde{V}\tilde{V}} = \sigma^2Q_{LL}U^T(UQ_{LL}U^T)^{-1}UQ_{LL} \quad (17)$$

The a posteriori variance factor of the estimated

$$\text{weight unit: } \hat{\sigma}^2 = \frac{\tilde{V}^T P \tilde{V}}{m} \quad (18)$$

Let 's define the following operator matrix:

	$m$	$n$
$n$	$U^T$	$I$
$1$	$F$	$L$

After transformation, we obtain by applying (1), (2), (3) and (4):

$$\begin{aligned} W_1 &= U^T R^{-1} \\ W_2 &= I - U^T R^{-1} (U^T R^{-1})^T I \\ W_3 &= F R^{-1} \\ W_4 &= L - F R^{-1} (U^T R^{-1})^T I \end{aligned}$$

By explaining these relations, we obtain the following results:

$$\begin{aligned} W_2 &= I - U^T R^{-1} R^{-1T} U \quad \text{with } R^{-1T} = R^{T-1} \\ W_2 &= I - U^T (R^T R)^{-1} U \\ W_2 &= I - U^T (UW_1 W_1^T U^T)^{-1} U \\ W_2 &= I - U^T (U U^T)^{-1} U \end{aligned}$$

$W_2 = Q_{LL}$  the cofactor matrix a posteriori of the observations

$$W_4 = L - F R^{-1} (R^{-1T} U)$$

$$W_4 = L - F (R^T R)^{-1} U$$

$$W_4 = L - F (UW_1 W_1^T U^T)^{-1} U$$

$$W_4 = L - F (U U^T)^{-1} U$$

$W_4 = \tilde{L}$  the vector of adjusted observations

We have an additional relation:  $W_1 W_3^T = \tilde{L} = W_4$

We find after transformation, the following matrix:

	$m$	$n$
$n$	$W_1$	$Q_{LL}$
$1$	$W_3$	$L$

If we want to take into account the variance-covariance matrix of the a priori observations, we must apply the same remark as in 3.3.1 by factorizing the variance-covariance matrix and arranging the operator matrix as follows:

	$m$	$n$
$n$	$U^{T*}$	$Q_{LL}$
$1$	$F$	$L$

$Q_{LL}$  and (D) right (G) triangular matrix :

$$Q_{LL} = GD = GG^T = D^T D$$

We will pose as before:  $U^{T*} = D^T U^T$

## Rank deficiency, generalized inverse and free network

So far we have assumed that systems of linear equations are composed of independent column vectors, without linear combinations. However, in the case of free networks where no point is fixed, the solution  $\tilde{X} = (A^T P A)^{-1} (A^T P L)$  no longer exists because the determinant  $|A^T P A|$  will be close to zero. The solution consists in calculating a pseudo-inverse or generalized inverse. Generalized orthogonalization provides an elegant solution.

Suppose therefore that the matrix of the coefficients of the observation equations  $A$  contains  $r$  linearly independent columns and the rest  $d$  of linearly dependent columns.

Without losing the generality, we can assume that the linearly dependent columns are located in the right part of the matrix  $A$ . We denote the part of  $A$  which contains linearly independent columns  $A_1$  and that contains of linearly dependent columns  $A_2$  and the matrix of their linear combination  $\alpha$ .

$$A = (A_1, A_2) \text{ and } A_2 = A_1\alpha \text{ with } X = \begin{pmatrix} X_1 \\ X_2 \end{pmatrix} \quad (18)$$

We can now rewrite the solution as:

$$V = A_1X_1 + A_2X_2 - L = A_1(X_1 + \alpha X_2) - L = A_1\tilde{X} - L \quad (19)$$

Since the matrix  $A_1$  contains no linearly dependent columns, a unique solution for  $\tilde{X}$  exists that minimizes the Euclidean norm of  $V$ .

If we know the matrix  $\alpha$  and the vector  $\tilde{X}$  then any solution  $X$  of

$$\tilde{X} = X_1 + \alpha X_2 = (1, \alpha)X \quad (20)$$

Will also be the least squares solution of (20) with the same vector of residuals  $V$ . Let's see how the generalized orthogonalization algorithm handles this case.

	$m-rd$	$rd$	$1$	$m-rd$	$rd$	$1$
$n$	$A_1$	$A_2$	$-L$	$Q$	$0$	$V$
$m-rd$	$I$	$0$	$0$	$R_1^{-1}$	$-\alpha$	$X$
$rd$	$0$	$I$	$0$	$0$	$I$	$0$

With  $rd$  (rank deficiency) which can be calculated using Givens rotations.

The first orthogonalization is carried out on the matrix  $A_1$

If we wish to obtain only the set of solutions for the parameters,  $X$  a second orthogonalization will be carried out on:

	$rd$	$1$
$m-rd$	$-\alpha$	$X_1$
$rd$	$I$	$0$

$$\text{To resolve : } \begin{bmatrix} -\alpha \\ I \end{bmatrix} X_2 = \begin{bmatrix} X_1 \\ 0 \end{bmatrix} \quad (21)$$

If on the contrary, we wish to obtain the whole of the solutions for the parameters  $\tilde{X}$  but also the matrix of cofactors  $Q_{\tilde{X}\tilde{X}}$

the second orthogonalization will be carried out on the matrix operator following with permutation of the sub-matrices  $-\alpha$  and  $R_1^{-1}$  and their corresponding lower ones.

	$m-rd$	$rd$	$1$
$m-rd$	$-\alpha$	$R_1^{-1}$	$X_1$
$rd$	$I$	$0$	$0$

The developments that we presented in 3.3.1 and 3.3.2 also apply here and we refer the reader to references [12] and [13] and to the numerical example.

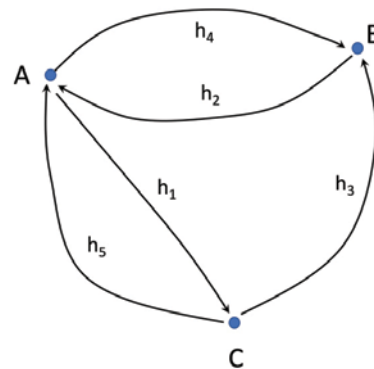
## Numerical example

We propose in this paragraph a numerical example, both to illustrate our presentation and to offer readers wishing to test their own software a numerical basis of comparison.

### Leveling Network

To illustrate the duality between the two models (Gauss-Markov and Gauss Helmert), we will handle that example by the observation equations and the condition equations.

Consider the leveling network represented by the following diagram:



Only point A is known in altitude and  $Z_A = 124.18m$

We have  $h_1$  the following values for the elevations:

$$\begin{aligned} h_1 &= 6.14 \text{ m} \\ h_2 &= 8.34 \text{ m} \\ h_3 &= -14.48 \text{ m} \\ h_4 &= -8.35 \text{ m} \\ h_5 &= -6.16 \text{ m} \end{aligned}$$

The distance between points A, B and C will be equal to unity to simplify the stochastic model and we only deal with 2 decimals for the observations.

## Observation equation

We can express the functional relations that exist between the altitudes of points A, B and C and the value of the differences in height  $h_1$ :

$$\begin{aligned} Z_B &= Z_A + h_1 \\ Z_A &= Z_C + h_2 \\ Z_C &= Z_B + h_3 \\ Z_C &= Z_A + h_4 \\ Z_A &= Z_B + h_5 \end{aligned}$$

The linear functional model will be written:

$$\begin{aligned} V_1 + h_1 &= Z_B - Z_A \\ V_2 + h_2 &= -Z_C + Z_A \\ V_3 + h_3 &= Z_C - Z_B \\ V_4 + h_4 &= Z_C - Z_A \\ V_5 + h_5 &= -Z_B + Z_A \end{aligned} \quad (22)$$

Which in matrix form is written  $L + V = AX + A_0$ :

$$\begin{bmatrix} h_1 \\ h_2 \\ h_3 \\ h_4 \\ h_5 \end{bmatrix} + \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \\ -1 & 1 \\ 0 & 1 \\ -1 & 0 \end{bmatrix} \cdot \begin{bmatrix} Z_B \\ Z_C \end{bmatrix} + \begin{bmatrix} -Z_A \\ Z_A \\ 0 \\ -Z_A \\ Z_A \end{bmatrix}$$

The matrix operator is presented as follows, with the matrix result after orthogonalization:

$$A = \begin{bmatrix} 1 & 0 & -130.32 \\ 0 & -1 & 115.84 \\ -1 & 1 & 14.48 \\ 0 & 1 & -115.83 \\ -1 & 0 & 130.34 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \quad W = \begin{bmatrix} 0.577 & 0.204 & 0.006 \\ 0.000 & -0.612 & 0.001 \\ -0.577 & 0.408 & -0.007 \\ 0.000 & 0.612 & 0.009 \\ -0.577 & -0.204 & 0.014 \\ 0.577 & 0.204 & 130.326 \\ 0.000 & 0.612 & 115.839 \end{bmatrix}$$

With the different sub-matrices of  $W$ :

$$W_1 = \begin{bmatrix} 0.577 & 0.204 \\ 0.000 & -0.612 \\ -0.577 & 0.408 \\ 0.000 & 0.612 \\ -0.577 & -0.204 \end{bmatrix} \quad W_2 = \begin{bmatrix} 0.006 \\ 0.001 \\ -0.007 \\ 0.009 \\ 0.014 \end{bmatrix} \quad W_3 = \begin{bmatrix} 0.577 & 0.204 \\ 0.000 & 0.612 \end{bmatrix}$$

$$W_4 = \begin{bmatrix} 130.326 \\ 115.839 \end{bmatrix}$$

The parameters being the altitudes of points B and C, we obtain:

$$\begin{aligned} Z_B &= 130.326 \text{ m} \\ Z_C &= 115.839 \text{ m} \end{aligned}$$

The cofactor matrix of the adjusted observations will be:

$$Q_{\bar{L}\bar{L}} = W_1 W_1^T$$

$$Q_{\bar{L}\bar{L}} = \begin{bmatrix} 0.375 & -0.125 & -0.250 & 0.125 & -0.375 \\ -0.125 & 0.375 & -0.250 & -0.375 & 0.125 \\ -0.250 & -0.250 & 0.500 & 0.250 & 0.250 \\ 0.125 & -0.375 & 0.250 & 0.375 & -0.125 \\ -0.375 & 0.125 & 0.250 & -0.125 & 0.375 \end{bmatrix}$$

The variance-covariance matrix of the adjusted observations will be:

$$C_{\bar{L}\bar{L}} = \sigma^2 Q_{\bar{L}\bar{L}} \text{ with } \sigma^2 = \frac{w_2^T w_2}{n-m}$$

The variance-covariance matrix of the adjusted parameters will be obtained as follows:

$$C_{\bar{X}\bar{X}} = \sigma^2 (W_3^T W_3)$$

We obtain the a posteriori standard deviations by taking the square root of the diagonal elements of the matrix  $C_{\bar{X}\bar{X}}$ :

$$\sigma_{z_B} = \sqrt{C_{\bar{X}\bar{X}(1,1)}} = \pm 0.006 \text{ m}$$

$$\sigma_{z_C} = \sqrt{C_{\bar{X}\bar{X}(2,2)}} = \pm 0.007 \text{ m}$$

## Conditional equations

We can write the following linear relations to express the conditions imposed on the observations  $h_1$ :

$$\begin{aligned} h_1 + h_2 + h_3 &= 0 \\ h_1 + h_5 &= 0 \\ h_2 + h_4 &= 0 \end{aligned}$$

Which in matrix form is written  $UL + U_0 = F$ :

$$\begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} 6.140 \\ 8.340 \\ -14.480 \\ -8.350 \\ -6.160 \end{bmatrix} = \begin{bmatrix} 0.000 \\ -0.020 \\ -0.010 \end{bmatrix}$$

$$\text{With } U_0^T = [0 \quad 0 \quad 0]$$

The operator and result matrices look like this:

$$A = \begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0.00 & -0.02 & -0.01 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$W = \begin{bmatrix} 0.577 & 0.516 & -0.158 & 0.375 & -0.125 & -0.250 & -0.125 & -0.375 \\ 0.577 & -0.258 & 0.474 & -0.125 & 0.375 & -0.250 & -0.375 & 0.125 \\ 0.577 & -0.258 & -0.316 & -0.250 & -0.250 & 0.500 & 0.250 & 0.250 \\ 0.000 & 0.000 & 0.791 & 0.125 & -0.375 & 0.250 & 0.375 & -0.125 \\ 0.000 & 0.775 & 0.158 & -0.375 & 0.125 & 0.250 & -0.125 & 0.375 \\ 0.000 & -0.015 & -0.011 & 0.006 & 0.001 & -0.007 & 0.009 & 0.014 \end{bmatrix}$$

One notes the perfect similarity between the model of the observation equations and the model of the conditioned equations with regard to the numerical results obtained for  $\bar{L}$  and  $Q_{\bar{L}\bar{L}}$ .

## Rank deficiency, generalized inverse and free network

In the example of the leveling network, let's ignore the dimension of point A. This is a free network adjustment (without constraint) whose interest is to focus on the observations. One could say that the model with condition equations also fulfill this role

except that the automation of the closures is not easy. As such, the model with observation equations is easier to program since for each observation we have an equation.

In our example leveling network, we only have one known point in elevation and we can refer to this adjustment as a minimum constraint adjustment.

The functional model is written as follows, considering the altitude of point A as a parameter to be estimated:

$$\begin{bmatrix} h_1 & V_1 & -1 & 1 & 0 \\ h_2 & V_2 & 1 & 0 & -1 \\ h_3 + V_3 & 0 & -1 & 1 & 1 \\ h_4 & V_4 & -1 & 0 & 1 \\ h_5 & V_5 & 1 & -1 & 0 \end{bmatrix} \begin{bmatrix} Z_A \\ Z_B \\ Z_C \end{bmatrix}$$

The rank deficiency can be calculated using Givens rotations and in our example it will be 1. The operator matrix is written:

$$A = \begin{array}{c} 5 \\ 2 \\ 1 \end{array} \begin{array}{c} m-rd=2 \\ rd=1 \\ 1 \end{array} \begin{array}{|c|c|c|c|} \hline -1 & 1 & 0 & 6.14 \\ \hline 1 & 0 & -1 & 8.34 \\ \hline 0 & -1 & 1 & -14.48 \\ \hline -1 & 0 & 1 & -8.35 \\ \hline 1 & -1 & 0 & -6.16 \\ \hline \hline 1 & 0 & 0 & 0 \\ \hline 0 & 1 & 0 & 0 \\ \hline 0 & 0 & 1 & 0 \\ \hline \end{array}$$

With the result matrix:

$$W = \begin{array}{c} 5 \\ 2 \\ 1 \end{array} \begin{array}{c} m-rd=2 \\ rd=1 \\ 1 \end{array} \begin{array}{|c|c|c|c|} \hline -0.500 & 0.354 & 0.000 & 0.006 \\ \hline 0.500 & 0.354 & 0.000 & 0.001 \\ \hline 0.000 & -0.707 & 0.000 & -0.008 \\ \hline -0.500 & -0.354 & 0.000 & 0.009 \\ \hline 0.500 & -0.354 & 0.000 & 0.014 \\ \hline \hline 0.500 & 0.354 & 1.000 & 8.341 \\ \hline 0.000 & 0.707 & 1.000 & 14.488 \\ \hline 0.000 & 0.000 & 1.000 & 0.000 \\ \hline \end{array}$$

It can be seen that the vector of residuals is identical to that obtained by the other models (observation equations and conditioned equations).

Moreover, and as mentioned before, the singular column relates to the last parameter which is the dimension of Point C. We can indeed re-arrange the parameters and make this singularity relate to the parameter of our choice.

We will proceed to the second orthogonalization which will relate to the lower part of the matrix operator is:

$$\begin{array}{c} m-rd \\ rd \end{array} \begin{array}{|c|c|c|} \hline m-rd & rd & 1 \\ \hline -\alpha & R_1^{-1} & X_1 \\ \hline 1 & 0 & 0 \\ \hline \end{array}$$

Either by considering the results of the first orthogonalization:

$$W = \begin{array}{c} 3 \\ 3 \end{array} \begin{array}{c} m-rd=2 \\ rd=1 \\ 1 \end{array} \begin{array}{|c|c|c|c|} \hline 0.500 & 0.354 & 1.000 & 8.341 \\ \hline 0.000 & 0.707 & 1.000 & 14.488 \\ \hline 0.000 & 0.000 & 1.000 & 0.000 \\ \hline R_1^{-1} & -\alpha & & \\ \hline \hline m-rd=2 & rd=1 & 1 & \\ \hline 1.000 & 0.500 & 0.354 & 8.341 \\ \hline 1.000 & 0.000 & 0.707 & 14.488 \\ \hline 1.000 & 0.000 & 0.000 & 0.000 \\ \hline -\alpha & R_1^{-1} & & \end{array}$$

After permuting the sub-matrices  $-\alpha$  and  $R_1^{-1}$ , we proceed to the second orthogonalization to obtain:

$$W = \begin{array}{c} 3 \\ 3 \end{array} \begin{array}{c} m-rd=2 \\ rd=1 \\ 1 \end{array} \begin{array}{|c|c|c|c|} \hline 0.577 & 0.333 & 0.000 & 0.732 \\ \hline 0.577 & -0.167 & 0.354 & 6.878 \\ \hline 0.577 & -0.167 & -0.354 & -7.610 \\ \hline X & & & \end{array}$$

We now have solutions for points A, B and C which correspond to a mean plane which has replaced l'altitudepoint A as in the previous example. This is the typical result of a generalized inverse and illustrates the resolution of a free network.

We can check the results obtained by considering again the altitude from point A to deduce the altitude of points B and C.

$$Z_B = Z_A + 6.878 - 0.732 = 130.326 \text{ m}$$

$$Z_C = Z_A - 7.610 - 0.732 = 115.838 \text{ m}$$

The matrix of the cofactors of the parameters being written  $Q_{\bar{x}\bar{x}} = WW^T$ :

$$Q_{\bar{x}\bar{x}} = \begin{bmatrix} 1.000 & 0.000 & 0.000 \\ 0.000 & 0.167 & 0.000 \\ 0.000 & 0.000 & 0.250 \end{bmatrix}$$

This result further illustrates one of the attractions of free network adjustment since the parameter cofactor matrix is devoid of correlation.

$$C_{\bar{x}\bar{x}} = \sigma^2 Q_{\bar{x}\bar{x}}$$

And we obtain for the parameters the following a posteriori standard deviations:

$$\sigma_{z_A} = \sqrt{C_{\bar{x}\bar{x}(1,1)}} = \pm 0.016 \text{ m}$$

$$\sigma_{z_B} = \sqrt{C_{\bar{x}\bar{x}(2,2)}} = \pm 0.007 \text{ m}$$

$$\sigma_{z_C} = \sqrt{C_{\bar{x}\bar{x}(3,3)}} = \pm 0.008 \text{ m}$$

Free network adjustment allows to conduct error detection tests, such as those we use and which are based on Professor Baarda's B method [36], and also to test the congruence of control points.

This is the typical approach used for deformation networks in digital monitoring where the stability of the control points must also be tested before performing a constraint adjustment.

## Algorithm

The programming of the generalized orthogonalization algorithm does not offer any particular difficulty and we give an example written in ANSI C language.

As input, we need a matrix  $A$  whose dimensions are as follows:

$$A = \begin{array}{c} \begin{array}{cc} m & r \\ \hline n & \begin{array}{|c|c|} \hline A_1 & A_2 \\ \hline \end{array} \\ \hline p & \begin{array}{|c|c|} \hline A_3 & A_4 \\ \hline \end{array} \end{array} \end{array} \quad W = \begin{array}{c} \begin{array}{cc} m & r \\ \hline n & \begin{array}{|c|c|} \hline W_1 & W_2 \\ \hline \end{array} \\ \hline p & \begin{array}{|c|c|} \hline W_3 & W_4 \\ \hline \end{array} \end{array} \end{array}$$

and whose result will replace its initial content.

```
gso(c,m,r,n,p)
double c[N+1][N+1];
int m,r,n,p;
{
    int x,i,j,h;
    double s,prd;
    for(h=1;h<=m;h++)
        {
            s=0.0;
            for(x=1;x<=n;x++)
                {
                    s = s + c[x][h] * c[x][h];
                    for(j=1;j<=n+p;j++)
                        c[j][h] = c[j][h] / sqrt(s);
                    for(i=h+1;i<=m+r;i++)
                        {
                            prd=0.0;
                            for(x=1;x<=n;x++)
                                prd = prd + c[x][h] * c[x][i];
                            for(j=1;j<=n+p;j++)
                                c[j][i] = c[j][i] - c[j][h] * prd;
                        }
                }
        }
}
```

transformation numbers

scalar product

normalization

transformation

## Conclusion


At the end of this article, we hope that the reader has been able to take into consideration the beauty and elegance of this algorithm. The conciseness of this one and the fact that the transformations are carried out in the initial matrix operator, allows us to use it abundantly in all the software applications that we develop where we have to apply the method of least squares adjustment and the method B test invented by Professor Baarda of Delft [15].

The generalized orthogonalization does not require the formation of the normal equations and is therefore numerically more stable than the conventional method [14]. In addition, it can handle all equation models as well as the adjustment of free networks that are common in monitoring and deformation study projects.

We would like to thank Aleš Čepeck from the Czech Technical University in Prague for our recent discussions and exchanges on generalized orthogonalization. He was the collaborator of F. Charamza and one of the authors of the free software GNU GAMA which is dedicated to the adjustment of topographic networks. This software also uses generalized orthogonalization.

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## Dataplör expands in 100 countries

Dataplör started in 2017 with a mission to map the local businesses of Mexico. Having rapidly expanded in the past year and on track to triple annual revenue, the company's data now covers 100 countries and 150 million POIs. It plans to extend coverage to 150 countries by the end of 2022 to continue providing customers with accurate POI data in developing economies. While pursuing growth, the firm remains committed to maintaining accuracy across all of its locations. By working with local experts to vet geospatial data, the firm closes gaps in machine errors with boots-on-the-ground verification to ensure its numbers truly reflect local businesses, which are always changing. This dynamic accuracy gives dataplör's intelligence an edge over rivals who have accurate information for developed economies like the US but haven't done the legwork to understand poorly mapped business geographies in, say, India or Brazil. *dataplör.com*

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## New Esri initiative provides free Geospatial software for nonprofits

Nonprofits are increasingly taking a geographic approach to implementing their plans and ensuring the success of their goals. To support and advance the missions of small nonprofits that may think GIS software is out of reach, Esri has launched the Esri Small Nonprofit Organization Grant Initiative. The grant initiative will provide 150 501(c)(3) nonprofit organizations with access to GIS training and software.

The GIS technology available to grantees will empower nonprofits of all sizes to better communicate their cause, understand communities, act on their mission, measure impacts, as well as extend services, attract volunteers, expand donor networks, and shape public policies. *www.esri.com*

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## India's Crop Map at Village-Level

RMSI Cropalytics, a Noida-based agri-tech start-up, launched the first of its kind crop map for India at the village level. The new crop map shows the geo-location of sown acreage of the current cropping season in high resolution.

Currently, crop acreages are estimated at a coarse resolution at the state or district level but not marked or verified on the map. The first-of-its-kind crop map will be updated at the village level every Kharif and Rabi season, covering major crops such as paddy, soybean, maize, sugarcane and wheat at high resolution. RMSI Cropalytics makes the crop map actionable by overlaying useful data on it, including village boundaries, farm sizes, crop health, weather forecasts, yield estimates and land ownership. It allows for more accurate acreage estimation by village and sharper identification of agrarian distress, as well as data-based decision-making on the deployment of resources, risk mitigation and procurement. *www.rmsi.com*



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## Bluesky wins extension to multi million pound contract

Aerial mapping company and consortium lead Bluesky International, in partnership with Getmapping plc, has been granted a contract extension from the Geospatial Commission to continue to supply aerial photography, 3D height models and Colour Infrared imagery, to public sector organisations across Great Britain. The contract, branded as Aerial Photography Great Britain (APGB), was first signed in 2018 enabling 4500 UK local authorities, emergency services, environmental bodies and central government departments to access, free at point of use, high quality aerial data. [www.bluesky-world.com](http://www.bluesky-world.com)


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## DWR launches new web-based mapping tool

A new web-based tool developed by the Department of Water Resources (DWR) will allow the public to explore thousands of groundwater projects across California to get a better understanding of one of the state's most critical water supply resources. The virtual mapping tool is part of the State's ongoing commitment to develop new, innovative solutions to provide information and resources to address the effects of California's changing climate and ongoing severe drought. The web-based tool is intended for anyone interested in learning more about state and local investments in groundwater sustainability and the return on those investments. <https://water.ca.gov>

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## Global Mapper Mobile expands GNSS device communication

Blue Marble Geographics has announced that Global Mapper Mobile version 2.3 provides additional options for connecting with GNSS receivers. It has always included the ability to connect to external GPS devices, but version 2.3 expands this to include support for any GNSS device that uses TCP/IP communication. Additionally, this mobile update provides Global Mapper analysis tools for volume calculation and viewed for terrain data. <http://bluemarblegeo.com> 

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## Government of Ukraine to have access to ICEYE SAR satellite constellation

ICEYE has signed a contract with the Serhiy Prytula Charity Foundation, which will provide the Government of Ukraine with its Synthetic Aperture Radar (SAR) satellite imaging capabilities. In addition, it will provide access to its constellation of SAR satellites, allowing the Ukrainian Armed Forces to receive radar satellite imagery on critical locations with a high revisit frequency. [www.iceye.com](http://www.iceye.com)

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## Pixxel's hyperspectral satellite technology for Australian farmers

Pixxel has partnered with Australian cloud-based agritech company DataFarming. Using Pixxel's hyperspectral dataset, DataFarming will be able to monitor crop health at new speeds and greater resolutions compared to the multispectral imaging on behalf of tens of thousands of farmers. The images allow for in-depth analysis of plant and soil biophysical and biochemical properties, allowing farmers to track these properties over the course of the growing season to improve their crop performance. [www.datafarming.com.au](http://www.datafarming.com.au)

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## Space Norway to build radar satellite for real time maritime surveillance

On the 25th of August 2022 Space Norway AS signed contracts with vendors and will immediately start building a radar satellite system optimized for maritime surveillance in Norwegian areas of interest.

The satellite system named MicroSAR is unique in the way it can detect relatively small vessels in a very large area simultaneously. As of today we don't know about any radar satellite systems with the same mix of capabilities.

Norway's sea areas are seven times larger than the Norwegian land area. The Arctic and the High North is Norway's most important strategic area of interest. This puts strong requirements on situational awareness in these areas. AIS

(Automatic Identification System) has for many years been used for maritime surveillance. The challenge is that AIS is a system that requires the vessels themselves to send the required and correct AIS Information. Hence, AIS is a system based on cooperation. Today we estimate that 5 % of the vessels either does not send out AIS Information or are transmitting false information. Satellites with a radar, such as MicroSAR, will be able to detect these vessels independent of the use of AIS. MicroSAR satellites will bring an AIS Receiver to correlate radar detections with AIS Information.

Space Norway works closely with the Norwegian Armed Forces who will be the main customer and buy services and products from MicroSAR when in operation. [www.sstl.co.uk](http://www.sstl.co.uk)

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## BlackSky awarded \$1.7m NASA contract to advance Earth Science research

BlackSky Technology Inc. has received its first call order, worth \$1.7 million, from NASA to evaluate accessibility, accuracy, quality and utility of the Company's imaging data services for the Commercial Smallsat Data Acquisition (CSDA) Program.

This call order was issued under a five-year Commercial Smallsat Data Acquisition Program, sole-source blanket purchase agreement announced in November 2021 to provide high-revisit satellite imaging data in support of NASA's existing Earth System Science and applications development for the benefit of society. [www.blacksky.com](http://www.blacksky.com)

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## Ursa Space Systems enter into pact to provide geospatial intelligence in Ukraine

Ursa Space will provide analytic services to EOS Data Analytics (EOSDA) in support of missions in Ukraine. The unique service built atop Ursa's services and EOSDA's capabilities will enable the delivery of critical insights over the country. [www.ursaspace.com](http://www.ursaspace.com)

## South Korea's first space launch to the moon

The Korea Pathfinder Lunar Orbiter (KPLO) lifted off on August 4 atop a SpaceX Falcon 9 rocket, kicking off South Korea's first-ever deep-space mission and setting the stage for more ambitious moon efforts down the road. KPLO, also known as Danuri, "will be the first step for ensuring and verifying [South Korea's] capability of space exploration," officials with the Korea Aerospace Research Institute (KARI), which is managing the mission, said in a statement.

This first step will lead toward a robotic moon landing by 2030, if all goes according to plan — a milestone that will be huge for South Korea. "Lunar exploration will enhance the space technologies of Korea, increase the value of Korea and stimulate pride [in] Korean[s]," the KARI statement added. KPLO's lunar arrival will come about a month after that of NASA's tiny CAPSTONE probe, which launched in late June and is taking a similarly circuitous path to Earth's nearest neighbor. [www.space.com](http://www.space.com)

## Hydrosat secures NOAA license

Hydrosat has secured a license agreement with National Oceanographic and Atmospheric Administration (NOAA) to operate a private remote sensing space system. The authorization represents a significant milestone for the fast-growing startup as it grants Hydrosat the requisite licensing to capture and sell data to commercial and government clients, and provides regulatory support as the company continues to make progress towards the launch of its innovative VanZyl-1 satellite mission.

## Russia launches Iranian satellite

A high-resolution Iranian-owned satellite has been launched into space from a base in Kazakhstan on board a Russian rocket. The remote-sensing Khayyam satellite, which Iran has said it wants to use for non-military purposes. [www.aljazeera.com](http://www.aljazeera.com)

## AWS Region in the UAE

The AWS Region in the United Arab Emirates is now open. The official name is Middle East (UAE), and the API name is me-central-1. One can start using it to deploy workloads and store your data in the United Arab Emirates. The AWS Middle East (UAE) Region is the second Region in the Middle East, joining the AWS Middle East (Bahrain) Region. <https://aws.amazon.com>

## Sanborn expands the Sanborn M-Map coverage

The Sanborn Map Company, Inc. has announced it has successfully completed the collection of geospatial data for 4,250 line miles of highway throughout the western United States, allowing the firm to appreciably expand the reach of its Sanborn M-Map® product line. Sanborn M-Map®, an HD Map product line developed for the autonomous vehicle market, provides precision datasets with absolute accuracy – thus enhancing safety and operational capability for our clients. [www.sanborn.com](http://www.sanborn.com)

## New tech to maritime infrastructure asset inspectors

Qii.AI has entered into a strategic alliance with Kongsberg Maritime, a provider of maritime technologies that is majority owned by the Norwegian government. Under the terms of the arrangement, sonar inspection experts and other subscribers to Kongsberg's recently launched ScanFuse™ service, will gain access to Qii.AI's asset visualization and inspection platform that enables collaborative, remote, AI-assisted inspection of concrete and steel structures like bridges, dams, ships, and turbines. <https://qii.ai>

## Resecurity expands AI-driven threat intelligence in Peru

Resecurity, Inc., a U.S. cybersecurity and intelligence company, announced its partnership with CFBD to expand Resecurity's AI-driven cybersecurity solutions and services to Peru. Based in

Lima, it is a leading distribution company specializing in engineering development and IT infrastructure for electronic security and artificial intelligence. Resecurity's innovative cybersecurity solutions allow organizations to automate the identification, assessment, and triage of incoming cyber threats while staying ahead of cybercriminals using advanced tactics to attack organizations at scale. <https://cfbd.co>

## Pony.ai and SANY to develop next-gen autonomous trucks

Pony.ai has announced a strategic joint venture with SANY Heavy Truck (SANY), a subsidiary of SANY Heavy Industry, China's number one heavy equipment manufacturer and the third largest heavy equipment manufacturer globally. The two companies will deeply integrate Pony.ai's "virtual driver" with SANY's technical accumulation in the field of wire-controlled chassis and vehicle development to jointly develop high-end heavy trucks that are automotive-grade and have L4-class redundancies. <https://pony.ai>

## DeepRoute.ai completes test of its autonomous driving solution

DeepRoute.ai has announced the results of the latest fully-driverless test of its Driver 2.0 L4 production-ready autonomous driving solution. It released a video exhibiting a driverless vehicle retrofitted with a production-ready L4 solution on Central Business District roads in Shenzhen, demonstrating its advanced capacity in complex and challenging traffic environments. [DeepRoute.ai](https://deerpoute.ai)

## Lyft unveils self-driving cars in Las Vegas

US-based ride-hailing service business Lyft has said that it will allow its users to hail a new kind of self-driving vehicle around the Las Vegas Strip. Significantly, the car will have two 'safety drivers,' who will override the control in case of any emergency or other discrepancies. This scenario puts the entire dialogue and hype around autonomous driving into question



as the technology required for self-driving cars still lags behind. Even after spending a decade promising self-driving cars, tech and automotive companies fail to deliver the promise. Moreover, the driverless autonomous car is still years, even decades behind entering into commonplaces, according to The New York Times.

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### Arm, Cruise and the driverless road ahead

Britain-based semiconductor maker Arm announced its partnership with US-based self-driving company Cruise LLC to catapult the development of self-driving autonomous vehicles. This partnership lets Arm develop high-performance, low-power computing systems while letting innovators like Cruise continue and experiment with its innovations. [www.arm.com](http://www.arm.com)

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### Ecom Express partnership with what3words

Ecom Express, one of India's leading last-mile delivery service providers, has announced its partnership with innovative location technology what3words to ensure that deliveries can be made precisely to any 3-metre square in India using just three words. Now, Ecom Express' customers (online shopping sites) can add a what3words field at checkout to ensure precise address and a best-in-class delivery experience for end-consumers.

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### Autonomous vehicle navigation with mapping solution

Sandvik Mining and Rock Solutions has introduced the AutoMine® Mapping Solution to maximise productivity and improve safety of autonomous vehicle navigation in underground mining operations through the use of mapped data.

It is a next generation product that enables a vehicle to safely record an underground 3D environment with a mine mapping tool, and convert 3D maps to 2D. Faster configuration, and the possibility to continue to operate other equipment within the area while it is being mapped, increases productivity and efficiency. ▴

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### New drone detection technology could lead to safer airspace

The Ohio Department of Transportation and Federal Aviation Administration have teamed up to create a technology that "detects and avoids" drones from colliding with other low-flying aircraft such as medical helicopters or crop dusters. As the number of private drones rises, the concerns about how to keep airspace safe for everyone rise as well.

The Uncrewed Traffic Management system features three ground-based radar locations in central Ohio. For drone users, operators must request clearance to fly and use the airspace from the FAA, similar to how a commercial pilot would before take-off. The system would also allow for more drones to be used in emergency situations, relaying valuable information to crews on the ground.

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### Commaris announces partnership with FIZUAS

Commaris™, a brand of Terrafugia, Inc. announced its dealer partnership with FIZUAS Unmanned Aircraft Systems. This partnership allows FIZUAS to sell Commaris's flagship long-range UAV, the SEEKER, for commercial use. The Seeker's innovative, electric, fixed-wing/VTOL hybrid aircraft is designed to perform a wide variety of commercial inspection operations, including security and surveillance, inspections, surveying and mapping, and more. [www.Commaris.com](http://www.Commaris.com)

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### Next gen medical drones for deployment in Ukraine

Draganfly Inc. has delivered next-generation Medical Response Drones to Coldchain Delivery Systems, LLC for provision of its highly specialized next-generation Medical Response Drones to Revived Soldiers Ukraine ("RSU"). These Drones has been equipped with custom hardware and software updates that include an automatic payload drop function that uses proximity sensors for safe cargo release from several feet above

the ground, a significantly expanded communication range, and enhanced first-person viewing capabilities that allow for a day or night operations. [www.draganfly.com](http://www.draganfly.com)

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### DedronePortable for military and commercial entities

Dedrone has launched DedronePortable, an all-in-one kit that offers comprehensive drone detection, tracking, and identification capabilities (DTI) on the go. It offers machine learning technology and end-to-end defeat capabilities via a system that can be set up in the field in less than 20 minutes. It can be combined with DroneDefender to not only DTI enemy/unauthorized drones but also deny and disrupt the RF bands used by most commercial and military drones, and across the GPS, GLONASS, BeiDou, Galileo, SBAS, and QZSS geo-location bands. [www.dedrone.com](http://www.dedrone.com)  
Test site selected for FAA UTM field test

The Federal Aviation Administration (FAA) has awarded a task order contract to the New York UAS Test Site (NYUASTS) for the next uncrewed aircraft systems (UAS or drone) integration project to help advance the reality of safe, high-volume drone operations, and the development of a UAS traffic management (UTM) system. The UTM Field Test (UFT) project will be managed by NUAIR on behalf of the NYUASTS located in Oneida County, NY and conducted within the 50-Mile Drone Corridor between Rome and Syracuse. The project will provide the FAA and industry with information to support policy development and will help industry update standards to support beyond visual line-of-sight (BVLOS) operations – a critical element in unlocking the true potential of routine, commercial drone operations. <https://nuair.org>

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### Indian Army signs pact with Drone Federation

The Army Design Bureau on behalf of the Indian Army has signed an MoU

**EASA's C2 certificate for eBee X Series drones**

senseFly, announced that eBee X, eBee Geo and eBee Ag are the first commercial drones to be designated with the C2 class identification label in accordance with the European Aviation Safety Agency (EASA) regulations. As of August 22, 2022, drone operators flying C2 labeled eBees will be able to conduct missions in the "Open Category", with all the advantages that this entails.

The C2 certification allows the eBee X series, with correct labelling, to fly at a horizontal distance of 30 meters from uninvolved people. By contrast, heavy drones like VTOLs or quadcopters must maintain a distance of 150 meters from people and any residential, commercial, industrial and recreational areas, limiting their operational capabilities to remote zones. [www.sensefly.com](http://www.sensefly.com)

**T-Mobile's network connects window-cleaning drones**

T-Mobile for Business will provide the 4G LTE connectivity for Lucid Drone Technologies' fleet of industrial drones, which do cleaning for commercial building surfaces and windows.

It will deliver 4G LTE connectivity to Lucid Drone to facilitate real-time sharing of flight data, battery utilization information, hardware diagnostics, and delivery of firmware and software updates. Intermap & Lufthansa Systems join OneSky Future of flight program

Intermap Technologies together with Lufthansa Systems has joined the OneSky Future of Flight program. It aims to help ecosystem partners to navigate the challenges of integrating their technology with air traffic management solutions. Intermap and Lufthansa Systems will combine their powerful terrain data, including vegetation and buildings, with OneSky's airspace management system. The integrated technology

with the Drone Federation of India to collaboratively work towards promoting research, development, testing and manufacturing of drones, counter-drone and associated technologies that can assist the Indian Army in its operations.

The Army Design Bureau is the nodal agency of the Indian Army to be the facilitator for the R&D efforts with the Industry, Academia, DRDO and DPSUs to enable them to understand and appreciate user requirements in depth.

**Garuda Aerospace ventures into Africa**

Chennai, India based drone startup Garuda Aerospace has partnered with Harare Institute of Technology and Nyangani Virtual University to offer drones-as-a-service to the agriculture industry in Zimbabwe. This is Garuda's second global venture and the first in Africa. In June, Garuda announced that it is setting up a drone factory in Malaysia in collaboration with HiiLSE Drones, a Malaysian drone company.

**Drones to monitor 250 acre floating solar farm in Thailand**

Percepto has successfully completed a proof-of-concept (POC) with the Electric Generating Authority of Thailand (EGAT) for the monitoring of a 250-acre floating solar farm, one of the largest of its kind in the world.

The size of 70 soccer fields and located 350m from the nearest shoreline, the solar farm is on the Sirindhorn dam basin in Southeastern Thailand, consisting of seven sections floating on buoys. Partnering with Top Engineering Corporation, a Thai drone consultancy and equipment provider, Percepto AIM software and Percepto Air drone-in-a-box will enable autonomous routine inspections of panels and other equipment to ensure proper operation and detect anomalies before they turn into bigger issues.

Joining Thailand's electrical grid last October, the \$34 million project reflects the country's push to achieve carbon neutral status by 2050 with 145,000 solar panels harnessing power from the sun during the day and converting energy from flowing water at night.

Given the solar farm's size and distance from land, equipment inspection and maintenance are cumbersome. Without drones, inspection staff would need to access the panel by boat to manually review the panels or go offshore to simply launch a drone that could provide visual inspection. Inspectors also face obstacles due to weather conditions such as extreme heat, rain and fog.

The automated drones will provide regular operations and maintenance reports, map the location of the panel, and perform inspections of substations, transformers, floating fences and solar floaters, which hold the panel above water. When an anomaly is detected, workers can know the exact problem, and where it is located. While one person must be in the vicinity of the panel for regulatory reasons, the inspection can be conducted remotely from Bangkok, more than 600 km away from the dam. [www.percepto.co](http://www.percepto.co)

**HAL developing long-endurance drone to up vigil over LAC**

State-run aerospace behemoth Hindustan Aeronautics Ltd (HAL) is working on an AI-driven multi-role, advanced and long-endurance drone for strategic missions in high-altitude areas including along the frontiers with China, people familiar with the development said.

The rotary-wing drone will have the capability to carry a load of 40 kgs, including missiles and sensors, and it is being developed considering the requirement of the armed forces to keep a strong vigil over the mountainous areas along the Line of Actual Control (LAC), they said. [www.business-standard.com](http://www.business-standard.com)

will empower users with best-of-breed data and analytics to enhance flight safety. [www.intermap.com](http://www.intermap.com)

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### Next-gen precision farming system by Trimble and CLAAS

As part of a strategic alliance, Trimble and CLAAS have developed a next-generation precision farming system for CLAAS tractors, combines, and forage harvesters. The precision farming system includes the new CLAAS CEMIS 1200 “smart” display, GPS PILOT steering system, and the SAT 900 GNSS receiver. The CEMIS display utilizes Trimble’s new embedded modular software architecture for positioning, steering, and ISOBUS technology for a seamless connection to control and monitor implements in the field. [www.trimble.com](http://www.trimble.com)

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### Geneq Inc releases the SXblue SMART

Geneq Inc. has just released the SXblue SMART to meet the requirements of professionals looking for accuracy and flexibility for field work. It features a GNSS engine capable of tracking all-in-view GNSS signals with high speed, saving time on the field. In addition, its mechanism and antenna design are at the forefront of interference mitigation technology and optimize its ability to handle a wide frequency band. [Geneq.com](http://Geneq.com)

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### Maxar to build 14 spacecraft platforms for L3Harris

Maxar Technologies has been selected by L3Harris Technologies for the design and production of 14 spacecraft platforms and associated support for its Tranche 1 Tracking Layer contract with the Space Development Agency (SDA).

SDA selected L3Harris as a prime for its Tranche 1 Tracking Layer as part of the initial Missile Warning/Missile Tracking warfighting capability of the National Defense Space Architecture (NDSA). It will provide limited global indications, warning and tracking of conventional and advanced

missile threats, including hypersonic missile systems. [www.maxar.com](http://www.maxar.com)

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### New Mesa Pro Rugged Tablet

Juniper Systems has announced its entry in the 10-inch rugged tablet segment with the introduction and launch of the all-new Mesa Pro Rugged Tablet. Featuring 11th Generation Intel Core processors, a Windows 11 operating system, device customization options, a large sunlight-readable display, and Juniper Rugged design, the Mesa Pro will be the most robust 10-inch rugged tablet on the market. [www.junipersys.com](http://www.junipersys.com)

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### Antenova adds Agosti corner-placement antenna

Antenova Ltd is adding a new offering to its range of miniature Surface Mount Designed (SMD) antennas and modules for GNSS applications. The new antenna, Agosti, part number SR4G080, measures 9.0 x 5.8 x 1.7 mm and operates with exceptional efficiency in a reduced space on a corner of a PCB.

The key advantage of the antenna is its small ground plane requirement. [www.antenova.com](http://www.antenova.com)

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### Trimble RTX Integrity validates positioning data accuracy

Trimble has introduced data integrity monitoring for its precise point positioning (PPP) correction service, CenterPoint® RTX Fast. The Trimble RTX Integrity™ monitoring system is an innovative, patented solution, built in direct response to client requirements for production-ready applications. It continuously validates the reliability of correction data processed by the network, which is broadcast to users in the agriculture, geospatial, construction and automotive industries, ensuring positioning data is right the first time.

Through a unique two-step process, the Trimble RTX Integrity system verifies the integrity of GNSS data received from satellites and filters faulty information in the network server before the data is

broadcast. A secondary post-broadcast check is conducted on the entire data transmission process where additional errors may be detected and removed. The integrity monitoring system is fully automated and reacts in seconds to detect, isolate and block faulty data to provide even more highly accurate and reliable positioning. [www.trimble.com](http://www.trimble.com)

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### Nextech AR launches AR Wayfinding

Nextech AR Solutions Corp. has launched a major new technology advancement with AR wayfinding into the events spaces as well as upgrades to its augmented reality spatial mapping platform, “ARway.” ARway and Map D, a self-serve event management software solution, are now offering clients a total package event management product that now includes AR wayfinding, opening up another major potential revenue driver. [www.arway.ai](http://www.arway.ai)

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### RS2+ GNSS receiver with LTE modem

Emlid has launched the Reach RS2+ GNSS receiver. An evolution of Emlid’s popular flagship RS2 multi-band receiver, it has a built-in 4G LTE modem and features a range of design improvements to enhance Reach usability in the field. It has been developed based on surveyor feedback to further improve reliability, effectiveness and robustness. [heliguy™](http://heliguy™) has partnered with Emlid to offer its affordable and highly-accurate GNSS solutions which provide professional-grade positioning information to empower land surveying and drone mapping workflows. [heliguy™](http://heliguy™) in-house specialists can offer technical and workflow support to integrate Emlid solutions into clients’ operations. [www.heliguy.com](http://www.heliguy.com)

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### Unicore releases GNSS RTK module

Unicore Communications has released its new generation of high-precision GNSS module. The UM980 uses real-time kinematic (RTK) technology to achieve centimeter-level positioning accuracy. It is based on the small high-performance system-on-chip NebulasIV, which integrates radio frequency,

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### October 2022

**Geo For Good Summit 2022**

4-6 October

Mountain View, CA, USA

**UN ICG - 16th Meeting**

9 - 14 October 2022

Abu Dhabi, UAE

<https://space.gov.ae>

**2<sup>nd</sup> United Nations World Geospatial Information Congress (UNWGIC)**

10-14 October 2022

Hyderabad, India

<https://ggim.un.org/2unwgic>

**The 7<sup>th</sup> Geospatial Conference**

**The 6<sup>th</sup> SMPR and 4<sup>th</sup> GIREsearch**

15-18 October 2022

Tehran, Iran

<https://geospatialconf2022.ut.ac.ir>

**Intergeo Hybrid**

18-20 October 2022

Essen, Germany

[www.intergeo.de](http://www.intergeo.de)

### November 2022

**Trimble Dimensions 2022**

7-9 November

Las Vegas, USA

<http://dimensions.trimble.com>

### February 2023

**GeoWeek 2023**

13-15, February

Denver, CO, USA

[www.geo-week.com](http://www.geo-week.com)

### March 2023

**DGI 2023**

27 Feb-01 March

London, UK

<https://dgi.wbresearch.com>

### April 2023

**GISTAM 2023**

25-27 April

Prague, Czech Republic

<https://gistam.scitevents.org/Home.aspx>

### May 2023

**Geo Business 2023**

17-18 May

London, UK

[www.geobusinessshow.com](http://www.geobusinessshow.com)

### June 2023

**TransNav 2023**

21-23 June

Gdynia, Poland

<https://transnav2023.umg.edu.pl>

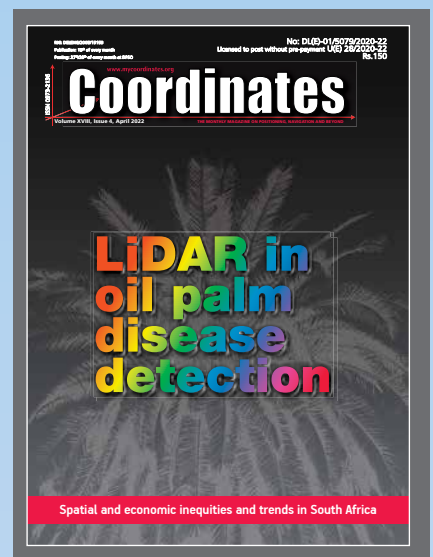
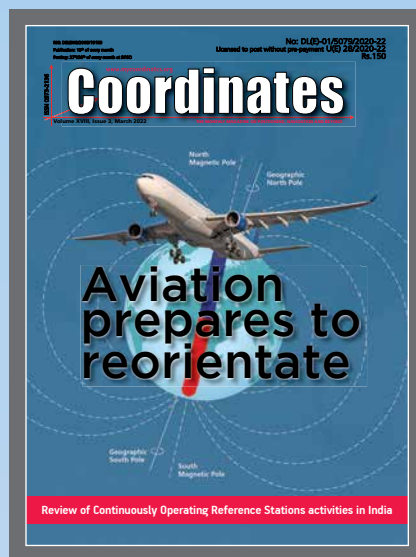
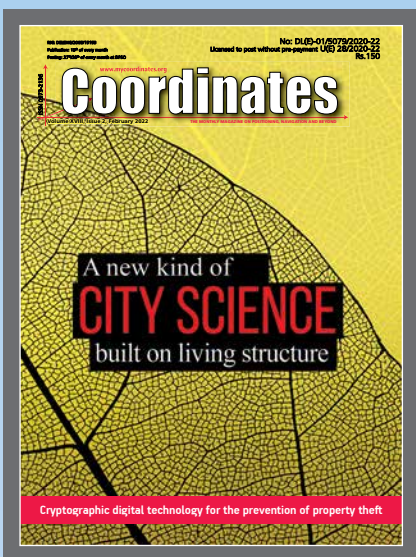
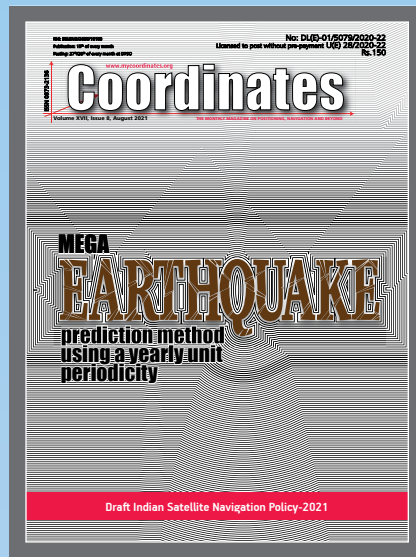
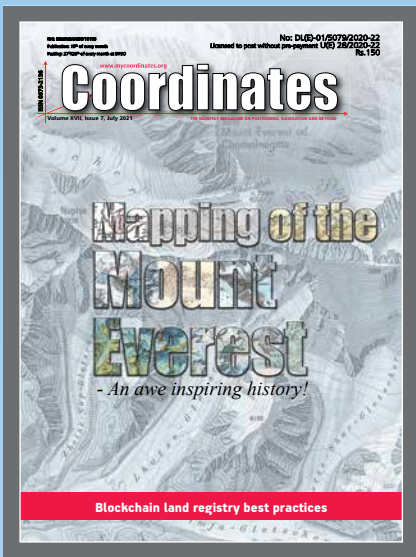
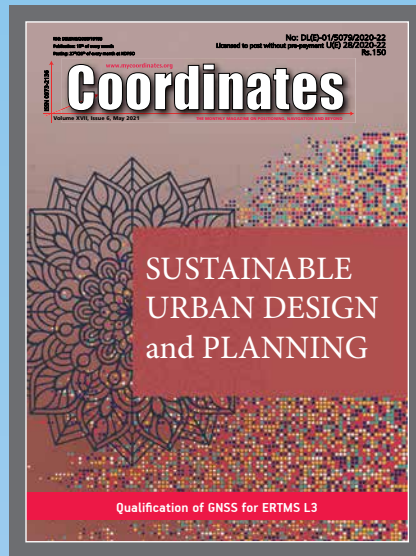
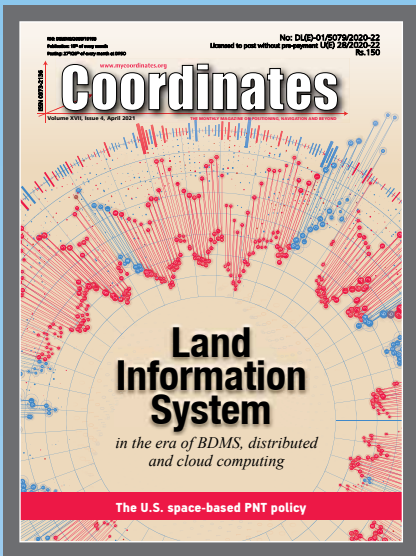
baseband and high-precision algorithms on a single chip. It has 1,408 channels to concurrently receive satellite signals from multiple constellations and multiple frequencies. <https://en.unicorecomm.com>

## NHAI organises workshop on GNSS based tolling

National Highway Authority of India organised a day-long stakeholder consultation workshop on Global Navigation Satellite System (GNSS) based tolling in India. The objective of the workshop was to seek input and suggestion from various industry experts and stakeholders on different aspects of the GNSS-based tolling system. The workshop will help strategise and design the future roadmap for free-flow tolling system in India based on GNSS technology.

Various stakeholders including relevant government departments, ISRO, NIC, and industry players such as Vehicle Manufacturers, AIS-140 On-Board Unit (OBU) manufacturers, Global GNSS service providers, Banks, Payment Aggregators/Gateway Service providers were invited for deliberations on GNSS based tolling system. Sessions and presentations were held with consultants & industry experts on global best practices for GNSS based tolling, discussions were held on the findings of the pilot carried out by NHAI on Delhi – Mumbai corridor and various aspects of GNSS architecture, payment settlement process, proposed enforcement measures and discussion on legal framework requirements were also discussed.

In the proposed GNSS technology-based tolling system, the NH stretch will be geo-fenced and shall consist of virtual tolling points. Whenever a vehicle fitted with GNSS OBU shall pass through this virtual tolling points, information on distance travelled will be calculated based on satellite signals from multi-constellation such as NavIC, GPS etc. and applicable fee shall be computed by the central GNSS software system and deducted from the users' bank account linked to the OBU. <https://pib.gov.in>



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